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Chromatic word-quasisymmetric functions of matroids

Billera, Jia, and Reiner (2009) introduced the quasisymmetric functions of matroids and showed that this defines a Hopf alegebra homomorphism which is a valuative invariant, i.e., isomorphic matroids have the same quasisymmetric function and polytopal decompositions of matroid base polytopes define relations among the corresponding quasisymmetric functions. In this project we study an analogue in non-commuting variables, the word-quasisymmetric functions. To every matroid M we associate a word-quasisymmetric function $\psi(M)$ and call this the chromatic word-quasisymmetric functions of a matroid. Matroids and word-quasisymmetric functions form Hopf algebras, and our map ψ between them is a homomorphism. We want to study the kernel, equivalently the image, of the map ψ from matroids to word-quasisymmetric functions, that is, we would like to understand which matroids are indistinguishable by the chromatic word-quasisymmetric functions. The map ψ is not an invariant, but we can show that it is valuative. Using Schubert matroids and nested matroids, special classes of matroids, we prove a lower bound of $2^d - d$ for the rank of the map ψ from matroids to the chromatic word-quasisymmetric functions in degree d and conjectured the upper bound of $d!$ is tight. Recent AI supported work shows that the conjecture is true. This is joint work with Raul Penaguiao.