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Four-Functions Inequalities, Skeps, and Schur Positivity

In 2007, Lam, Postnikov and Pylyavskyy proved that $s_\lambda s_\mu \leq_s s_{\lambda\vee\mu}\, s_{\lambda\wedge\mu}$, where $f \leq_s g$ means that $g-f$ is Schur positive, and conjectured more generally that $s_\lambda s_\mu \leq_s s_{\lambda'} s_{\mu'}$ whenever $\lambda'+\mu' = \lambda+\mu$ and the pair (λ',μ') is more balanced than (λ,μ). Speyer recently proved this conjecture using skeps, a new combinatorial model for Littlewood–Richardson coefficients. In another direction, Chan, Chen, Pak and Soskin proved a Schur-positive analogue of the Ahlswede–Daykin four-functions theorem, and conjectured a Schur-positive analogue of a midpoint inequality of Klartag and Lehec. We prove a single Schur-positive four-functions theorem that contains all of these: if nonnegative weights a, b, c, d on partitions satisfy $a(\lambda)\, b(\mu) \leq c(\lambda')\, d(\mu')$ whenever (λ',μ') is the image of (λ,μ) under a fixed certain pairing, then $\big(\sum_\lambda a(\lambda)s_\lambda\big)\big(\sum_\lambda b(\lambda)s_\lambda\big) \leq_s \big(\sum_\lambda c(\lambda)s_\lambda\big)\big(\sum_\lambda d(\lambda)s_\lambda\big)$. Suitable choices of the pairing and the weights give the Chan–Chen–Pak–Soskin inequality, Speyer's theorem, and the conjectured Schur-positive Klartag–Lehec inequality. This also applies to sign-normalized stable Grothendieck polynomials and to dual stable Grothendieck polynomials, giving four-functions inequalities in the Schur order for both families. This is joint work with Swee Hong Chan.