Applied and Computational Algebraic Geometry
Géométrie algébrique appliquée et calculatoire
(Org: **Taylor Brysiewicz** (University of Western Ontario), **Greg Smith** (Queen's University) and/et **Maksym Zubkov** (University of British Columbia))

HIROTACHI ABO, University of Idaho
Hunting for surfaces: a computer-aided approach to constructing surfaces in projective four-space

This talk explores a long-standing problem in classifying algebraic surfaces in projective four-space, inspired by the work of Ellingsrud and Peskine in the 1980s. I will review the classification problem and discuss recent progress, including our new findings. I will also introduce a companion Macaulay2 package developed to support these computations and to encourage further exploration of this classification problem. This is joint work with Kristian Ranestad and Frank-Olaf Schreyer.

KEVIN DAO, University of Wisconsin – Madison
Minimal Filling Architectures for Polynomial Neural Networks

Minimal filling architectures (MFAs) describe the smallest hidden-layer widths for which the neurovariety $\mathcal{V}_{\mathbf{d},r}$ of a polynomial neural network fills its ambient function space. The dimension of a neurovariety is an algebraic geometric measure of how expressive the neural network is. It was conjectured that every such architecture has unimodal hidden widths. In this talk, we describe counterexamples to this conjecture.

In the opposite direction, we report some results about MFAs related to this conjecture. Time permitting, we outline some computational results related to finding all MFAs efficiently, counts of MFAs, and other data. This is joint work (some of it in progress) with Jose Israel Rodriguez.

DAVID JOHNSON, The University of Western Ontario
Computing Smooth Connectivity with Routing Functions: Implementation and Applications

Routing functions, recently developed by Cummings, Hauenstein, Hong and Smyth, encode the smooth connectivity of real algebraic varieties. Upon fixing such a variety, one can define a routing function on it. The connected components of a graph associated to the critical points of the routing function are in bijection with the smoothly connected components of the variety. This correspondence also yields a membership test, determining whether a point lies on a given component. Furthermore, the Euler characteristic of each smoothly connected component can be computed from the routing function's critical points. Building on this work, we present a Julia package which implements these methods numerically. The critical points are computed with numerical path tracking in HomotopyContinuation.jl. We demonstrate the functionality of our package on several examples, including the computation of α-positive kinematic parameters of a Feynman diagram.

ELANA KALASHNIKOV, University of Waterloo

JAKE, LEVINSON, Université de Montréal
Gröbner degenerations of $\overline{M}_{0,n}$ and trees

The moduli space $\overline{M}_{0,n}$ has a natural embedding in a product of projective spaces, due to Keel and Tevelev, with very nice combinatorial and algebraic properties. In this talk, I will describe a family of especially well-behaved Gröbner degenerations of the moduli space. Specifically, these degenerations yield, in the limit, reduced unions of coordinate hyperplanes (i.e. squarefree monomial ideals), which can moreover be described and enumerated in a combinatorial manner. Our result recovers certain

known descriptions of the multidegrees of the embedding, and it also allows us to give minimal generators for the embedding's saturated multihomogeneous ideal. This is joint work with Tianyi Yu and Ian Cavey.

QIXIA LUO, University of British Columbia
Structural Identifiability in Non-Gaussian Graphical Discrete Lyapunov Models

Graphical discrete Lyapunov models are statistical models derived from first-order vector autoregressive (VAR(1)) processes, with applications to causal discovery when only single-time-point equilibrium data, rather than the full time-series trajectories, are available. Motivated by the classic theorem of Marcinkiewicz, which establishes that non-Gaussian distributions with cumulants of all orders must have non-zero cumulants at infinitely many orders, we investigate the additional identifying information when the distributions in these models are non-Gaussian. In particular, we explore how polynomial equations associated with the higher-order cumulants of distributions in this model can be leveraged to identify the underlying graph, or causal structure.

BRENT PYM, McGill University
Graph complexes and the motivic Galois group

Classical Galois theory studies the symmetries of solutions of polynomial equations. Motivic Galois theory extends this philosophy to symmetries of period integrals on algebraic varieties. It is typically quite difficult to compute the Galois conjugates of a given period integral. I will describe a simple, combinatorial algorithm to compute Galois conjugates for periods of the moduli space of marked curves of genus zero, via graph complexes. While the algorithm itself is quite concrete, the proof that it works involves some new ideas in logarithmic geometry and the theory of operads. This talk is based on forthcoming joint work with Clément Dupont and Erik Panzer.

GABRIEL RÉMOND-TIEDREZ, McGill University
Validating Polynomial Sum-Of-Squares Decompositions

Verifying nonnegativity of polynomials has applications throughout mathematics, from graph theory to optimization and dynamical systems. Certifying nonnegativity of a multivariate polynomial is NP-hard in general, but it can be made computationally tractable by imposing the more restrictive requirement that the polynomial admit a sum-of-squares (SOS) decomposition. This requirement can be equivalently posed as a semidefinite program (SDP), which can be solved numerically using interior-point methods (IPMs). However, SDP formulations of SOS programs are often ill-conditioned, a problem exacerbated by accumulated rounding errors. As a result, SDP solvers can return unreliable numerical approximations, sometimes even reporting feasible solutions to known infeasible problems. To increase trust in the numerical results, we present a one-step rigorous verification of SDP results for SOS programs. This improves upon previous results by incorporating interval arithmetic and state-of-the-art methods from computer-assisted proofs. We complement the theory with numerical demonstrations, ranging from simple certification of SOS representations of nonnegative polynomials to verifying the validity of hexahedral meshes for finite element methods. In this talk, I will compare our approach to existing methods and discuss the limitations that arise from low-quality numerical results, an issue common to all existing methods.

ÁLVARO RIBOT, Harvard University
Do neural models prefer their submodels?

A neural model is the set of functions that can be expressed by a given neural network architecture. Neural models contain natural submodels, such as those obtained by setting some neurons to be inactive. We study the geometry of neural models and ask whether they prefer their submodels. By this we mean whether local minimizers of the loss function lie in a given submodel. We introduce a geometric framework to determine if this occurs. For shallow multilayer perceptrons, we show that subnetworks with inactive hidden neurons are never preferred. For deep multilayer perceptrons, we give conditions under which subnetworks with inactive hidden neurons are not preferred. For lightning self-attention, lower-rank submodels are usually not

preferred, but we identify examples where they are because they lie on a boundary of the model. Our results show that, aside from a few exceptions, preference for submodels cannot be explained by the geometry of the neural model, and must instead come from other sources such as training dynamics. This is joint work with Yulia Alexandr and Anna Seigal.

MATT SATRIANO, University of Waterloo
On non-negativity of stringy Hodge numbers

Crepant resolutions have inspired connections between birational geometry, derived categories, representation theory, and motivic integration. Stringy Hodge numbers are invariants of log-terminal varieties designed to capture the Hodge theoretic information of a crepant resolution, even when no crepant resolution exists. In this talk, we will characterize log-terminal singularities as those admitting a crepant resolution by a stack. We will additionally give a complete classification for when stringy Hodge numbers are non-negative, including explicit counter-examples to Batyrev's non-negativity conjecture. This is based on joint work with Jiahui Huang and Jeremy Usatine.

EMMA SCHMIDT, University of Notre Dame
Real Monodromy Action and the 3RPR Mechanism

The algebraically-defined Galois group directly corresponds to the geometric monodromy group over the complex numbers, which studies how solutions permute along loops in parameter space. In applications, it is more useful to study the real monodromy action based on the behavior of tiered collections of real solutions. This talk will introduce an algorithm to compute real monodromy structures. As a practical example, we will explore the monodromy action of the kinematic 3RPR planar mechanism. This is a joint work with Jonathan Hauenstein, Margaret Regan, and Charles Wampler.

MATIJA TOMIC, University of British Columbia