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Validating Polynomial Sum-Of-Squares Decompositions

Verifying nonnegativity of polynomials has applications throughout mathematics, from graph theory to optimization and dynamical systems. Certifying nonnegativity of a multivariate polynomial is NP-hard in general, but it can be made computationally tractable by imposing the more restrictive requirement that the polynomial admit a sum-of-squares (SOS) decomposition. This requirement can be equivalently posed as a semidefinite program (SDP), which can be solved numerically using interior-point methods (IPMs). However, SDP formulations of SOS programs are often ill-conditioned, a problem exacerbated by accumulated rounding errors. As a result, SDP solvers can return unreliable numerical approximations, sometimes even reporting feasible solutions to known infeasible problems. To increase trust in the numerical results, we present a one-step rigorous verification of SDP results for SOS programs. This improves upon previous results by incorporating interval arithmetic and state-of-the-art methods from computer-assisted proofs. We complement the theory with numerical demonstrations, ranging from simple certification of SOS representations of nonnegative polynomials to verifying the validity of hexahedral meshes for finite element methods. In this talk, I will compare our approach to existing methods and discuss the limitations that arise from low-quality numerical results, an issue common to all existing methods.