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On Kuperberg's quadrilateral conjecture

Kuperberg conjectured in 1983 that every convex body K in the plane is contained in a quadrilateral of area at most $\frac{3}{\sqrt{5}}\,|K|$, the extremal bodies being the affine-regular pentagons. On the basis of numerical experiments we propose a stronger conjecture of a purely polygonal nature: every convex polygon P is contained in a quadrilateral of area at most $\frac{3}{\sqrt{5}}\,|P|$ whose sides are parallel to sides or diagonals of P. This chord conjecture implies the inequality conjectured by Kuperberg. For pentagons it is a theorem of Hong, Ismailescu, Kwak and Park, of which we give a short proof by area identities. We also show that the classical bound $\sqrt{2}$ remains valid for quadrilaterals with sides parallel to chords; the proof is similar to earlier proofs of this bound, but it starts from an inscribed quadrilateral of maximal area instead of a circumscribed quadrilateral of minimal area. We close with a discussion of the difficulties in reaching the constant $\frac{3}{\sqrt{5}}$.