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On the rank of random symmetric matrices

How likely is an $n \times n$ random symmetric matrix to have rank at most $n - k$? For $k = 1$, Campos, Jenssen, Michelen and Sahasrabudhe proved in a breakthrough paper that the singularity probability for symmetric Rademacher matrices is exponentially small. More recently, Han went beyond singularity by proving a large deviation inequality for the rank: in the symmetric subgaussian model, the probability of rank at most $n - k$ is bounded above by e^{-cnk} for $1 \leq k \leq c'\sqrt{n}$.

In this talk, I will present an extension of Han's result. Under weaker assumptions on the entry distributions, we obtain an upper bound of $2e^{-cnk}$ for the full range $1 \leq k \leq n$. For the adjacency matrix of the Erdős–Rényi random graph $G(n, p)$ with $p = p(n)$ uniformly bounded away from 0 and 1, the case $k = 1$ confirms a conjecture of Costello and Vu.

This is joint work with Huaijin Liang and Kexin Yu.