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Density bounds for connected components on the square grid

Consider the square grid ($\mathbb{Z}^2$) under the two natural notions of nearest-neighbor connectivity (4 and 8-connectivity). For any natural number k, what fraction of the grid can be covered by connected components of size at most k? We give a simple exact answer in the 8-connected case, along with a natural construction that achieves it. In the 4-connected case, the situation is far more subtle; we construct a bound that is tight for at least $1/2$ of all natural numbers k. Our results further generalize to handle any prescribed set of connected components and rectangular subsets of the square grid. This improves upon prior work of Gravier and Payan (2001). Finally, we discuss a continuous analogue related to the Honeycomb theorem.