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Softness properties of tilings

We study various softness properties of tilings of $\mathbb{R}^d$ that are homeomorphic images of polyhedral tilings. Given a topological cell C of $\mathbb{R}^d$, a boundary point $x \in \partial C$ is called k-*soft* if ∂C locally contains a k-dimensional smooth manifold through x, while it is k-*sharp* otherwise. The main question we study is the following: for given $2 \leq d$ and $1 \leq k \leq d-1$, what is the infimum of the average number of k-sharp vertices per cell in homeomorphic images of sufficiently regular polyhedral tilings of $\mathbb{R}^d$? Domokos, G. Horváth and Regős showed that in dimension two, every such tiling has at least two 1-sharp vertices per cell on average, and this bound is sharp. In $\mathbb{R}^3$, the answer depends on the value of k. On the one hand, we prove that every normal, locally polyhedral tiling of $\mathbb{R}^3$ can be transformed into a *completely* 1-*soft* tiling. On the other hand, we show that the average number of 2-sharp vertices per cell must be at least $10/3$, and there exists a sequence of periodic examples approaching this bound. Our proof combines variants of Euler's formula for polyhedral complexes with the signature theorem of Izmestiev and Schlenker. The method also implies a sharp weak lower bound for the *harmonic degree* of the tilings, that was introduced by Domokos and Lángi.

This is a joint work with Dorottya Dancsó (University of Szeged).