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Random points in ball-convex bodies: Sylvester's problem and beyond

We study Sylvester's classical four-point problem for ball-convex bodies, where convex hulls are replaced by intersections of closed balls of fixed radius R. We show that the Efron–Buchta identities extend to this setting.

We determine the corresponding Sylvester probabilities for three and four uniform random points in an arbitrary planar ball-convex body. We also extend Groemer's inequality, showing that the expected area of the ball-convex hull is minimised by the disc of equal area.

For the unit disc, we obtain formulas involving dilogarithmic functions in terms of R, recover the classical Sylvester probabilities as $R \to \infty$, and derive the distribution function of the circumradius of three uniform random points on $[1, \infty)$.

Joint work with A. Bakó–Szabó (University of Szeged, Hungary) and F. Besau (Technical University of Vienna, Austria).