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Differentiability in homotopy theory

Homotopy theory is not usually seen as a setting for differentiation: homotopy classes of spaces and maps lack the rigidity needed for classical derivatives. Yet homotopy theory and calculus have been closely linked since Goodwillie's functor calculus of the 1990s, which shows that homotopy-invariant functors admit Taylor-like approximations whose layers behave like derivatives. Recent work with collaborators clarifies the structural basis for this analogy. With Johnson, Osborne, Riehl, and Tebbe, we showed that abelian functor calculus carries the structure of a *cartesian differential category*, providing an intrinsic notion of differentiation. With Burke and Ching, we further demonstrated that this viewpoint extends to the setting of homotopy functor calculus while simultaneously extending these ideas to infinity categories.

These ideas flow both ways: categorical structures illuminate the essential ingredients of functor calculus, while topological phenomena inspire new categorical constructions. Recent works of Schwarz in *tangent* categories, and work of Ching and Arro in *tangent infinity categories*, use these methods to generalize differential bundles, revealing new structures relevant to functor calculus.

This talk will survey these developments and outline emerging directions in differentiability and homotopy theory.