
AUSTIN SUN, University of Toronto

The Grassmannian of lines as the space of pencils of binary quantics: towards a GIT-free PGL_2 -stratification of $Gr(2, n+1)$

In the last century, geometric invariant theory (GIT) has been central to the study of reductive group actions on varieties such as Grassmannians. Using GIT, one may classify orbits of a reductive group acting on an algebraic variety and obtain stratifications of that variety consisting of collections of orbits. However, even with well-studied examples such as the n -dimensional projective space viewed as the space of binary quantics under the natural PGL_2 -action, we only know explicit stratifications for small n 's: as n grows larger, one would have to compute exceptionally large numbers of polynomial invariants for the PGL_2 -action in order to give an explicit stratification.

The goal of this talk is to illustrate an approach for constructing PGL_2 -stratifications of the Grassmannian $Gr(2, n+1)$ viewed as the space of pencils of binary quantics with minimal knowledge of GIT, which is the subject of one of the speaker's current research directions. Specifically, the proposed approach will not involve any computations of polynomial invariants for the PGL_2 -action, and will be based on the Hilbert-Mumford criterion, Schubert cell decompositions as well as the theory of algebraic groups. I will discuss some of the unresolved difficulties for writing down explicit PGL_2 -stratifications for $Gr(2, n+1)$ in general, and make connections to contemporary research on PGL_2 -stratifications for the projective space as the space of binary quantics.