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Ranking Quantilized Mean-Field Games and Early-Stage Venture Investments

We study a class of quantilized mean-field game models with a capacity for ranking games, where the performance of each agent is evaluated based on its terminal state relative to the population's α -quantile value, $\alpha \in (0, 1)$. This evaluation criterion is designed to select the top $(1 - \alpha)\%$ performing agents. We provide two formulations for this competition: a target-based formulation and a threshold-based formulation. In the former and latter formulations, to satisfy the selection condition, each agent aims for its terminal state to be *exactly* equal and *at least* equal to the population's α -quantile value, respectively.

For the target-based formulation, we obtain an analytic solution and demonstrate the ϵ -Nash property for the asymptotic best-response strategies in the N -player game. Specifically, the quantilized mean-field consistency condition is expressed as a set of forward-backward ordinary differential equations, characterizing the α -quantile value at equilibrium. For the threshold-based formulation, we obtain a semi-explicit solution and numerically solve the resulting quantilized mean-field consistency condition.

Subsequently, we propose a new application in the context of early-stage venture investments, where a venture capital firm financially supports a group of start-up companies engaged in a competition over a finite time horizon, with the goal of selecting a percentage of top-ranking ones to receive the next round of funding at the end of the time horizon. We present the results and interpretations of numerical experiments for both formulations discussed in this context and show that the target-based formulation provides a very good approximation for the threshold-based formulation.