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Regularity of maximal functions in higher dimensions

The classical Hardy-Littlewood maximal function theorem states that the Hardy-Littlewood maximal operator is a bounded operator on $L^p(\mathbb{R}^d)$ if and only if $1 < p \leq \infty$. In 1997 Juha Kinnunen proved the corresponding result for the gradient of the maximal function, i.e. that the $L^p(\mathbb{R}^d)$-norm of the gradient of the maximal function is controlled by the $L^p(\mathbb{R}^d)$-norm of the gradient of the function if $1 < p \leq \infty$. However, he provides no counterexample in the endpoint $p = 1$, and so in 2004 Hajłasz and Onninen formally posed the question if the endpoint gradient bound also holds.

Many special cases, generalizations and variations of this problem have been explored, with partial success. The original question by Hajłasz and Onninen remains unanswered. We discuss recent progress in higher dimensions, based on the coarea formula, dyadic decompositions and the relative isoperimetric inequality. As a by-product we obtain a Vitali-type covering lemma for the boundary.