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Muckenhoupt A_p weights, BMO, distance functions and Hardy-Sobolev inequalities

Vasin (for $n = 1$) and Anderson, Lehrbäck, Mudarra, and Vähäkangas (for $n > 1$) provided a geometric characterization of the sets $E \subset \mathbb{R}^n$ so that $w = dist(\cdot, E)^{-\alpha}$ is a Muckenhoupt A_1 weight for some $\alpha > 0$. We provide a geometric characterization of the sets $E \subset \mathbb{R}^n$ (which we call median porous sets) so that $w = dist(\cdot, E)^{-\alpha}$ is a Muckenhoupt A_p weight for some $\alpha > 0$ (given any $1 < p \leq \infty$).

Given $1 < p \leq \infty$, we also find the precise range of exponents α so that $w = dist(\cdot, E)^{-\alpha} \in A_p$ (in analogy to the $p = 1$ case done by Anderson, Lehrbäck, Mudarra, and Vähäkangas).

With our characterization we prove that $\mathbb{R}^n \setminus E$ supports a Hardy-Sobolev inequality if E is an appropriate median porous set. All previous such results that we are aware of make the strictly stronger assumption that the set E is porous. The proofs rely on a new median-value characterization of BMO. Joint work with Marcus Pasquariello.