SHIVARAM PRAGADA, Simon Fraser University
Structure of Eigenvectors of Graphs

Let G be a graph on n vertices with characteristic polynomial $\varphi_G(\lambda)$. A graph is said to be *Irreducible* if the characteristic polynomial of its adjacency matrix is irreducible. For every irreducible graph G, we show that each eigenvector of its adjacency matrix has pairwise distinct, non-zero entries.

More generally, consider a graph G whose characteristic polynomial factors over $\mathbb{Q}$ as

$$\varphi_G(\lambda) = p_1(\lambda) \cdots p_k(\lambda),$$

where the polynomials $p_i(\lambda)$ are distinct irreducible factors. For any eigenvalue θ with minimal polynomial $p_j(\lambda)$, we prove a structure theorem of eigenspaces corresponding each polynomial $p_j(\lambda)$. We derive a lower bound on the number of distinct entries that must appear in every eigenvector corresponding to θ.

It is conjectured that almost all graphs have irreducible characteristic polynomials, this has recently been confirmed under the assumption of the Extended Riemann Hypothesis. We pose new structural questions about irreducible graphs and present preliminary progress toward understanding their eigenvectors and spectral properties.