Variational Analysis: Theory and Applications
Analyse variationnelle : Théorie et applications
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ALEKSANDR ARAKCHEEV, The University of British Columbia
On Generalisations of Fejér Monotonicity: Fejér and Opial Sequences*

In this talk, we explore recently introduced extensions of Fejér monotonicity, namely Fejér* monotonicity and Opial sequences. We present a series of counterexamples showing that, in these generalized settings, several classical Fejér properties fail; nevertheless, in certain cases, suitable weakened variants of these properties can still hold. The connections between these notions and various forms of quasi-Fejér monotonicity are examined.

YUAN GAO, University of British Columbia Okanagan
On the equivalence of c-potentiability and c-path boundedness in the sense of Artstein-Avidan, Sadovsky and Wyczesany.

A cornerstone of convex analysis, established by Rockafellar in 1966, asserts that a set has a potential if and only if it is cyclically monotone. This characterization was generalized to hold for any real-valued cost function c and lies at the core structure of optimal transport plans. However, this equivalence fails to hold for costs that attain infinite values. In this talk, we explore potentiability for an infinite-valued cost c under the assumption of c-path boundedness, a condition that was first introduced by Artstein-Avidan, Sadovsky and Wyczesany. This condition is necessary for potentiability and is more restrictive than c-cyclic monotonicity. We provide general settings and other conditions under which c-path boundedness is sufficient for potentability, and therefore equivalent. We provide a general theorem for potentiability, requiring no topological assumptions on the spaces or the cost. We then provide sufficiency in separable metric spaces and costs that are continuous in their domain. Finally, we introduce the notion of a c-path bounded extension and use it to prove the existence of potentials for a special class of costs on $\mathbb{R}^2$. We illustrate our discussion and results with several examples.

HONGDA LI, University of British Columbia
Relaxed Weak Accelerated Proximal Gradient Method: A Unified Framework for Nesterov's Accelerations

This paper is devoted to the study of accelerated proximal gradient methods where the sequence that controls the momentum term doesn't follow Nesterov's rule. We propose a relaxed weak accelerated proximal gradient (R-WAPG) method, a generic algorithm that unifies the convergence results for strongly convex and convex problems where the extrapolation constant is characterized by a sequence that is much weaker than Nesterov's rule. Our R-WAPG provides a unified framework for several notable Euclidean variants of FISTA and verifies their convergences. In addition, we provide the convergence rate of the strongly convex objective with a constant momentum term. Without using the idea of restarting, we also reformulate R-WAPG as "Free R-WAPG" so that it doesn't require any parameter. Explorative numerical experiments were conducted to show its competitive advantages.

(Joint Work with Xianfu Wang.)

WALAA MOURSI,

SADRA NEJATI, University of British Columbia
Adjustable Robust Optimization Reformulations for Support Vector Machines

This work introduces new robust optimization reformulations for Support Vector Machines (SVMs) designed to make classification models more reliable when data are uncertain or noisy. We address uncertainty in both labels and features by combining ideas from adjustable and two- stage robust optimization. For label noise, our reformulation simplifies existing robust SVM models by cutting the number of binary variables nearly in half, leading to much faster com- putation without losing accuracy. For feature uncertainty, we propose a two-stage framework with a global uncertainty set that better reflects how data variability occurs in practice, solved efficiently using a Column-and-Constraint Generation algorithm. Across multiple benchmark datasets, our methods achieve up to twofold speed improvements and stronger robustness com- pared to traditional approaches, showing how optimization-based modeling can make machine learning systems both more efficient and more dependable in uncertain environments.

VIKTOR PAVLOVIK, University of Waterloo
Accelerated Proximal Gradient Methods in the affine-quadratic case

Recent works by Boţ-Fadili-Nguyen and by Jang-Ryu address the long–standing question of iterate convergence for accelerated (proximal) gradient methods. Specifically, Boţ- Fadili-Nguyen proved weak convergence of the discrete accelerated gradient descent (AGD) iterates and, crucially, convergence of the accelerated proximal gradient (APG) method in the composite case, in infinite–dimensional Hilbert spaces; their note also documents the an- nouncement timeline. In parallel, Jang-Ryu established point convergence both for the contin- uous–time accelerated flow and for the discrete AGD method in finite dimensions. These results leave unanswered the question of which minimizer is the limit point. We show in the affine-quadratic setting: starting from the same initial point, the difference between the PGM and APG iterates converges weakly to zero, so APG converges weakly to the best approximation of the starting point in the solution set; moreover, under mild conditions, APG converges strongly. Our results are tight: a two–dimensional example shows that this coincidence of limits is specific to the affine–quadratic regime and does not extend in general.

SHAMBHAVI SINGH, University of Waterloo
Eckstein-Ferris-Pennanen-Robinson duality revisited: paramonotonicity, total Fenchel-Rockafellar duality, and Chambolle-Pock

Finding zeros of the sum of two maximally monotone operators involving a continuous linear operator is a central problem in optimization and monotone operator theory. We revisit the duality framework proposed by Eckstein, Ferris, Pennanen, and Robinson from a quarter of a century ago. Paramonotonicity is identified as a broad condition ensuring that saddle points coincide with the closed convex rectangle formed by the primal and dual solutions. Additionally, we characterize total duality in the subdifferential setting and derive projection formulas for sets that arise in the analysis of the Chambolle-Pock algorithm within the recent framework developed by Bredies, Chenchene, Lorenz, and Naldi.

TUNG TRAN, UBCO
On the boundedness of sequences generated by stochastic gradient and random projection algorithms

We study the boundedness of sequences generated by two fundamental algorithmic frameworks: stochastic gradient methods and, as a special case, random projection algorithms. For the stochastic gradient method, we extend a result of Orvieto, Lacoste-Julien, and Loizou—originally established under strong convexity—to a broader class of functions, including coercive functions. In a complementary direction, we focus on random projection algorithms and generalize Meshulam's boundedness theorem from affine subspaces to polyhedral sets.