Theory and application of Inverse Problems in mathematical physics
Théorie et application des problèmes inverses en physique mathématique
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TRACEY BALEHOWSKY, University of Calgary
Transformation Optics and Models of Spatial Topology

What is the topology and geometry of the spatial universe? This fundamental question in cosmology remains open. Some cosmologists argue that closed 3-manifolds may provide a better fit to observational data than simply connected models. In this talk, I will describe how techniques from transformation optics allow one to physically model harmonic wave propagation on any closed, orientable 3-manifold.

First, I will give an overview of transformation optics. For our purposes, we prove a strengthened form of the Lickorish–Wallace link surgery theorem. Using this, we construct a diffeomorphism from the complement of a smooth link in a closed 3-manifold to a bounded domain in $\mathbb{R}^3$. Pushing forward the spatial metric through this map allows us to construct a metamaterial device whose induced anisotropic conductivity suitably reproduces the Helmholtz dynamics of the original manifold.

ALI FEIZMOHAMMADI,
Inverse spectral problems with sparse data and applications to photo-acoustic tomography

We discuss recent results on inverse spectral problems with sparse data on compact Riemannian manifolds with or without boundary. The goal is to reconstruct a general second order linear elliptic self-adjoint differential operator on a manifold from the knowledge of only a portion of its discrete spectrum along with the restrictions of an associated set of linearly independent eigenfunctions to a (possibly small) observation set. We show that such reconstructions are possible under certain geometric assumptions on the observation set as well as the differential operator. As an application of our inverse spectral result, we provide generic uniqueness results for the inverse photo-acoustic problem as well as other types of inverse problems with passive measurements where the goal is to determine the coefficients in a PDE along with the initial data/source term from one passive measurement of its solution on the observation set.

PETER GIBSON, York University
Inversion of the Miura map on the line

The Miura map relates the modified and classical Korteweg-de-Vries equations, as well as the one-dimensional Helmholtz and Schrödinger equations. The problem of inverting the Miura map arises in the context of scattering theory. Although the Miura map on the circle has long been known to be a global fold, the structure of the Miura map on the line has until recently been less well understood. In this talk we present a recent solution to the scalar Riccati equation that allows explicit inversion of the Miura map. We show that, for an appropriate notion of weak solution to the Riccati equation, inverse images of the Miura map on the line are parameterized by the Riemann sphere.

ISAAC HARRIS, Purdue University
Qualitative Methods Applied to Biharmonic Scattering

Inverse wave propagation problems arise in various fields, including non-destructive testing and medical imaging. The central challenge is to develop stable and reliable methods for identifying hidden obstacles or defects. This talk presents recent progress in extending qualitative reconstruction methods to biharmonic scattering problems, which describe wave scattering in long, thin elastic plates. This model is relevant to numerous engineering and physical systems.

Qualitative methods recover the shape of an unknown object from measured scattering data with minimal a priori information. However, these methods often break down at certain frequencies tied to an associated transmission eigenvalue problem. These eigenvalues can, in turn, serve as target signatures for estimating material properties, since they can be recovered from the scattering data and depend (often monotonically) on the unknown parameters.

The talk will outline new analytical results in qualitative reconstruction and explore their connection to transmission eigenvalue problems. Numerical methods for recovering the scatterer from the given data will also be discussed.

RU-YU LAI, University of Minnesota
Partial data inverse problems for the nonlinear magnetic Schrodinger equation

In this talk, we will discuss unique determination of the nonlinear coefficients in the dynamic nonlinear magnetic Schrodinger equation from the knowledge of the Dirichlet-to-Neumann map measured on an arbitrary part of the boundary. The main tool is the construction of special solutions to the linearized Schrodinger equation, which is obtained from the linearization process.

MICHAEL LAMOUREUX, University of Calgary
Inverse problems in seismic imaging

Mathematics has a long history of impact on the problem of seismic imaging, starting as far back as the early 1800's with the foundations of wave theory, acoustics, and of course the Fourier transform. This talk will present some of this history including early efforts in seismic measurements, tomography and reflection seismology, along with current developments in full waveform inversion and machine learning.

CRISTIAN RIOS, University of Calgary
Applications of Alpert wavelets to imaging-based medical diagnosis

We utilized an Alpert wavelet basis to encode images acquired as part of routine clinical late gadolinium enhancement (LGE) cardiac MRI protocols, and applied machine learning algorithms to develop a classification method to distinguish different types of cardio-myopathhies. The results were compared to non-preprocessed images and images processed with standard wavelet bases. In this talk we introduce the basic concepts of Alpert wavelets and some encouraging outcomes of the study. The research is done in collaboration with Ramneet Hunjan from the University of Calgary, and the Libin Cardiovascular Institute at the University of Calgary.

MAHISHANKA WITHANACHCHI, University of Calgary
Complex Analytic Methods in One Dimensional Scattering: Harmonic Exponentials, Inner Functions, and Toeplitz Kernels

One dimensional scattering for Schrödinger type and impedance form operators admits a rich complex analytic structure. In recent work, the scattering matrix can be written explicitly in terms of a harmonic exponential associated with the impedance profile, yielding closed form expressions for reflection and transmission coefficients as bounded analytic functions on the upper half plane. This description fits naturally into the framework of Hardy spaces, inner and outer factorization, and $SU(1,1)$ transfer matrices.

In this talk, I will explain how the transmission coefficient appears as an outer function in $H^\infty(\mathbb{C}_+)$ and how this connects to classical tools of complex analysis, including Poisson integrals, boundary uniqueness, and phaseless inverse problems. I will then discuss how the product integral formulation of the transfer matrix can be interpreted in the language of de Branges spaces and related to the Makarov and Poltoratski theory of meromorphic inner functions and Toeplitz kernels for canonical systems. This perspective suggests new ways to study completeness, spectral synthesis, and inverse scattering at low regularity by analyzing Toeplitz operators with symbols built from scattering data.

If time permits, I will outline ongoing work that uses these complex analytic techniques to formulate inverse and rigidity results for layered and low regularity media, emphasizing the role of Hardy space methods and the structure of inner and outer factors arising from the scattering matrix.