Recent progress in convex and discrete geometry
Progrès récents en géométrie convexe et discrète
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GERGELY AMBRUS, University of Szeged & Rényi Institute
Large signed sums, polarization problems, and projection constants of convex bodies

Vector balancing problems have been studied extensively for at least half a century, while their siblings, anti-balancing problems received much less attention. In this talk, we make up for this deficit by considering the following problem . Given a norm on $\mathbb{R}^d$, and a positive integer n, what is the largest constant $C(n)$ so that for any set of n unit vectors $u_1, \ldots, u_n$ in the chosen norm), one can assign signs $\varepsilon_i \in \pm 1$ to them for which the signed sum $\varepsilon_1 u_1 + \ldots \varepsilon_n u_n$ has norm at least $C(n)$? Equivalently, one would like to partition the set of vectors into two parts so that the corresponding sums are far from each other in the specified norm. We show that the worst case (i.e. the smallest $C(n)$) corresponds to the maximum norm, while $C(n)$ is asymptotically the largest for the Euclidean norm. En route, we also introduce the notion of the polarization constant of general convex bodies, demonstrate its connection to the question of large signed sums, and in the limit case, to the 1-absolutely summing and projection constants.

This is joint work with Florian Grundbacher (TU Munich).

KAROLY BEZDEK, University of Calgary
Non-separable arrangements revisited

A problem posed by Erdős in 1945 initiated the study of non-separable arrangements of convex bodies. A finite family of convex bodies in Euclidean d-space is called a non-separable family (or NS-family) if every hyperplane intersecting their convex hull also intersects at least one member of the family. Minimal coverings of NS-families of positive homothetic convex bodies have been investigated in several recent works. This lecture extends those results to weakly non-separable families and weakly k-impassable families of convex d-polytopes for $0 < k < d - 1$. This is a joint work with Z. Lángi (Rényi Inst. and Univ. of Szeged, Hungary).

TED BISZTRICZKY, University of Calgary
Construction methods for polytopes

A survey of the hows and whys of following methods of construction of d-dimensional convex polytopes:fundamental, inductive, descriptive and combinatorial.

DMITRY FAIFMAN, Université de Montréal
Bi-invariant valuations and convolution on Lie groups

Minkowski addition of sets is the geometric embodiment of the additive structure of the underlying Euclidean space. It gives rise to the convolution product on translation-invariant valuations, introduced by Bernig and Fu, which is closely tied to integral geometry, namely to additive kinematic formulas. Convolution of valuations has also been defined on compact Lie groups by Alesker and Bernig. We unify the two operations, introducing a convolution product of smooth bi-invariant valuations on arbitrary unimodular Lie groups. As a key ingredient in the construction, we find all connected Lie groups which admit non-trivial bi-invariant valuations. Based on a joint work with A. Bernig and J. Kotrbaty

FERENC FODOR, University of Szeged, Hungary
Series expansions for generalized random polygons, floating bodies and relative affine surface area

We prove power series expansions for random L-convex polygons in planar convex bodies with sufficiently smooth boundaries. Besides extending earlier asymptotic formulas, our results have consequences about L-convex floating bodies and relative affine surface area that have recently been investigated by Schütt, Werner and Yalikun (2025). This is joint work with D. Papvári (University of Szeged).

ALEX IOSEVICH, University of Rochester
Some parallels between Erdos type problems and exact signal recovery

We are going to discuss some connections between Erdos-type problems in the discrete setting and the theory of exact signal recovery. The restriction theory of the Fourier transform will be presented as the unifying theme.

JASKARAN KAIRE, University of Manitoba
Hadwiger's Conjecture for Cap Bodies

Hadwiger's covering conjecture states that every n-dimensional convex body can be covered by at most 2^n smaller positive homothetic copies of itself, with 2^n copies required only for affine images of the n-cube. Despite recent progress on the Hadwiger's conjecture, it remains open in general, as well as for specific classes of bodies.

In this talk, I will show that the conjecture holds for the class of cap bodies in all dimensions. For $3 \leq n \leq 15$, the proof combines a probabilistic technique with reduction to integer linear programming. For $n \geq 15$, we obtain an explicit bound based on the same probabilistic technique but avoiding computer aid. Furthermore, for $n = 3$, we prove that the value of the covering number for the class of cap bodies is 6.

This talk is based on joint work with A. Arman and A. Prymak.

PAVLOS KALANTZOPOULOS, University of Waterloo
Extremal Convex Bodies in Liakopoulos's Generalized Dual Loomis–Whitney Inequality.

We characterize equality cases in volume inequalities for sections of convex bodies by certain lower-dimensional linear subspaces. These inequalities, introduced by Liakopoulos, generalize Meyer's dual Loomis–Whitney inequality and provide a generalized dual form of the Bollobás–Thomason uniform cover inequality. Our approach builds on a previous result characterizing equality in Barthe's geometric reverse Brascamp–Lieb inequalities. We show that equalities occur when the convex body is the convex hull of its intersections with certain orthogonal subspaces determined by the Brascamp–Lieb data. This is joint work with Károly Böröczky and Ferenc Fodor.

SERGII MYROSHNYCHENKO, University of the Fraser Valley
Polytope Reconstruction: floating and illuminating structures

The surface of buoyancy (or surface of centers) of a convex body plays a central role in the study of floating bodies. Its properties and significance have been the focus of extensive recent work by D. Florentin, H. Huang, D. Ryabogin, C. Schutt, B. Slomka, E. Werner, B. Zawalski, N. Zhang, among others. In this talk, we address the problem of unique determination of a convex polytope by its surface of buoyancy or by its Dupin floating body. We also consider the dual questions in the setting of illumination bodies. The presented results are based on joint work with S. Dann and O. Herscovici.

LAM NGUYEN, Memorial University of Newfoundland
Logarithmic Sobolev, Poincaré, and Beckner Inequalities on Hyperbolic Spaces

This talk presents recent progress in the study of logarithmic Sobolev, Poincaré, and Beckner inequalities on hyperbolic spaces. We focus on determining the best constants and exploring their close connection to Gaussian measures. We also discuss new versions of Beckner inequalities that come from studying heat flow on hyperbolic spaces. This work is a collaboration with Anh Do, Guozhen Lu, and Debdip Ganguly.

EGON SCHULTE, Northeastern University
Bounding the Regularity Radius of Delone Sets

Delone sets are uniformly discrete point sets X in Euclidean d-space that are used in the modeling of crystals. They can be characterized by parameters r and R, where (usually) $2r$ is the smallest interpoint distance of X and R is the radius of a largest "empty ball" that can be placed into the interstices of X. The local theory for Delone sets searches for local conditions on X that guarantee the emergence of a crystallographic group of symmetries producing X as an orbit set consisting of a single point orbit or finitely many point orbits, respectively. The regularity radius $\hat{\rho}_d$ is defined as the smallest positive number ρ such that each Delone set X with congruent clusters of radius ρ is a regular system, that is, a point orbit under a crystallographic group. We discuss bounds for the regularity radius in terms of R, and present conjectures that have been verified for some particularly interesting classes of Delone sets. This is joint work with Nikolai Dolbilin, Alexey Garber and Marjorie Senechal.

CARSTEN SCHÜTT, Christian-Albrechts-University Kiel
Expected extremal area of facets of random polytopes

We study extremal properties of spherical random polytopes, the convex hull of random points chosen from the unit Euclidean sphere in $\mathbb{R}^n$. The extremal properties of interest are the expected values of the maximum and minimum surface area among facets.

KATERYNA TATARKO, University of Waterloo
Minimizing inradius for a given constraint

It is well known that among all convex bodies in $\mathbb{R}^n$ with a given surface area, the Euclidean ball has the largest inradius. We will show that this result can be reversed in the class of convex bodies with curvature at each point of their boundary bounded below by some positive constant λ (λ-convex bodies). In particular, we show that among λ-convex bodies of a given surface area, the λ-convex lens (the intersection of two balls of radius $\frac{1}{\lambda}$) minimizes the inradius. If time permits, we will also discuss a few related questions. This is a joint work with K. Drach.

VIKTOR VIGH, University of Szeged
Circumscribed random spherical disc-polygons via duality

In this talk, we work on S^2 and study random spherical disc-polygons within a spherical convex disc whose boundary is C^2 smooth. This model encompasses the usual spherical convex model and, as a limiting case, planar spindle convexity as well. We establish asymptotic results for the area and perimeter of the inscribed random disc-polygon. Next, we introduce a spherical spindle-convex duality, which allows us to naturally define circumscribed random spherical disc-polygons. Using the area and perimeter formulas of the dual disc, we obtain asymptotic results in the circumscribed model as well.

This is a joint work with Kinga Nagy.

ELISABETH WERNER,
L_p relative surface areas

Motivated by a duality result, we define new surface area measures for ball-convex bodies. We call these measures L_p relative surface areas. We show that these quantities are rigid motion invariant, upper semi continuous valuations. We establish monotonicity properties of these quantities which lead to new entropy functionals for convex bodies.

Joint work with D. Yalikun.

JIE XIAO, Memorial University
C^1-maximizer of p-mean torsion rigidity on convex bodies

Given a bounded domain $B \subset \mathbb{R}^{n\geq 2}$ with its boundary ∂B, a solution u_B of the torsion problem

$$\begin{cases} \Delta u_B = -1 & \text{in } B; \\ u_B = 0 & \text{on } \partial B, \end{cases}$$

is called a stress function of B. Via the torsion rigidity

$$\int_B |\nabla u_B(x)|^2 \, dx,$$

this talk is about to show that the maximization problem for $[1, \infty) \ni p$-mean torsion rigidity

$$(\star) \qquad \sup_{\text{all convex bodies } B \subset \mathbb{R}^n} \int_B \left(\frac{|\nabla u_B(x)|^2}{|B|^{\frac{2}{n}}} \right)^p \frac{dx}{|B|},$$

is achievable and the boundary $\partial B_\bullet$ of any maximizer $B_\bullet$ of $(\star)$ is C^1-smooth, thereby finding that if $|\nabla u_{B_\bullet}|$ is constant on $\partial B_\bullet$ then $B_\bullet$ is a Euclidean ball.

BARTLOMIEJ ZAWALSKI, Case Western Reserve University
On flat shadow boundaries from point light sources and the characterization of ellipsoids

The study of convex bodies whose hyperplane sections exhibit specific symmetry traces back to the classical works of W. Blaschke and H. Brunn. More recently, in a series of papers by I. Gonzalez-Garcia, J. Jeronimo-Castro, E. Morales-Amaya, and D.J. Verdusco-Hernandez, attention has turned to a dual viewpoint: instead of symmetries of sections by a prescribed family of hyperplanes, one investigates symmetries of supporting cones with apexes in a prescribed family of points. In this talk, we will present a precise duality framework that connects known and conjectured results in these two settings. We will further show that if the shadow boundaries generated by point light sources placed on a hypersurface enclosing a sufficiently smooth convex body $K \subset \mathbb{R}^n$, $n \geq 3$, are all flat, then K must necessarily be an ellipsoid. This extends a classical theorem of Blaschke, who established the analogous characterization in the case of parallel light sources placed at infinity.