Integrability, Geometry, and Symmetry of Differential Equations
(Org: **Stephen Anco** (Brock University) and/et **Konstantin Druzhkov** (University of Saskatchewan))

STEPHEN ANCO, Brock University

EVANS BOADI, State University of New York at Buffalo
Discrete Kutznetsov-Ma breather solutions of the focusing Ablowitz-Ladik equation

In this talk, I will discuss a class of solutions to the focusing Ablowitz–Ladik lattice, which are the discrete analogs of the Kutznetsov–Ma (KM) breathers of the focusing nonlinear Schrödinger equation. In 2015, the inverse scattering transform was used to construct a solution that was shown to be regular. In this talk, I will present a novel KM-type breather solution that is also regular on the lattice. Using Darboux transformation, I will also construct a multi-KM breather solution and demonstrate that the double KM breather remains regular on the lattice.

KOSTYA DRUZHKOV, University of Saskatchewan
Invariant reduction for partial differential equations: Poisson brackets

In this talk I will show that, under suitable conditions, finite-dimensional systems describing invariant solutions of PDEs inherit local Hamiltonian operators through a mechanism of invariant reduction, which applies uniformly to point, contact, and higher symmetries. The inherited operators endow the reduced systems with Poisson bivectors that relate constants of invariant motion to symmetries. The induced Poisson brackets agree with those of the original systems, up to sign. At the core of this construction lies the interpretation of Hamiltonian operators as degree-2 conservation laws of degree-shifted cotangent equations.

JORDAN FAZIO, Brock University
Hierarchies of Flow Invariants and Conservation Laws in One-Dimensional Fluids

Flow invariants are geometric quantities that are "frozen-in" to a fluid flow. They can tell us a lot about the properties of a system, such as integrability, and are closely related to the concept of conservation laws. The geometric character of flow invariants is general, and they can take the form of a scalar, vector, differential form, or a more general tensor. We start by introducing the structure of flow invariants in a general setting as well as their connection to conservation laws. We look at a relationship between invariants of different geometric character which enables us to construct a recursion operator acting on flow invariants. In one-dimensional fluids, we see how the recursion operator can produce infinite hierarchies of invariants, starting with two seed invariants given by the mass density and entropy of the fluid. This method is generalizable, and we look at how we can adapt this recursion operator on flow invariants to produce new conservation laws in isentropic one-dimensional fluid flows, using members of the well-known hierarchies of conservation laws as seeds for the generated hierarchies.

JAMES HORNICK, mcmaster
BIFURCATIONS OF SOLITARY WAVES IN A COUPLED SYSTEM OF LONG AND SHORT WAVES

We consider families of solitary waves in the Korteweg–de Vries (KdV) equation coupled with the linear Schrodinger (LS) equation. This model has been used to describe interactions between long and short waves. To get a comprehensive characterization of solitary waves, we consider a sequence of local (pitchfork) bifurcations of coupled solitary waves from the uncoupled KdV solitons. The first member of the sequence is the KdV soliton coupled with the ground state of the LS equation, which is proven to be the constrained minimizer of energy for fixed mass and momentum. The other members of the sequence are the

KdV soliton coupled with the excited states of the LS equation. We connect the first two bifurcations with the exact solutions of the KdV–LS system frequently used in the literature.

SERHII KOVAL, Memorial University of Newfoundland
Weyl algebras and symmetries of differential equations

Let $\mathbb{K}$ be a field of characteristic zero. The first Weyl algebra A_1 is a unital associative $\mathbb{K}$-algebra generated by elements x and ∂ that satisfy the defining relation $\partial x - x\partial = 1$. The nth Weyl algebra is the n-fold tensor product $A_1^{\otimes n}$, and it is canonically isomorphic to the ring of differential operators $\mathbb{K}[x_1, \dots, x_n][\frac{\partial}{\partial x_1}, \dots \frac{\partial}{\partial x_n}]$.

Weyl algebras are fundamental objects in ring theory and they arise in many branches of mathematics and physics, for example, quantum mechanics, representation theory and noncommutative geometry. In this talk, I will discuss how algebras A_n arise in symmetry analysis of differential equations, and what new knowledge about the structure of A_n can be obtained using symmetries of differential equations. This talk is based on a joint project with Roman O. Popovych.

MAHDIEH GOL BASHMANI MOGHADAM,
Symmetry Transformation Group Arising from the Laplace–Runge–Lenz Vector

The Kepler problem in classical mechanics exhibits a rich structure of conserved quantities, highlighted by the Laplace–Runge–Lenz (LRL) vector. Through Noether's theorem in reverse, The LRL vector gives rise to a corresponding infinitesimal dynamical symmetry on the kinematical variables, which are well known in the literature. However, the physically relevant part of the LRL vector is its direction angle in the plane of motion (its magnitude is just a function of energy and angular momentum).

In this talk, I will derive the infinitesimal dynamical symmetry corresponding to the direction part of the LRL vector, and obtain the explicit form of the symmetry transformations that it generates. When combined with the rotation symmetries, the resulting symmetry group is shown to be the semi-direct product of $SO(3)$ and R^3. This stands in contrast to the SO(4) symmetry group generated by the LRL symmetries and the rotations. As a by-product, the action of the new infinitesimal symmetries on all of the conserved quantities is obtained.

The results are given in terms of the physical kinematical variables in the Kepler problem, rather than in an enlarged auxiliary space in which the LRL symmetries are usually stated.

ALEXANDER ODESSKI, Brock University
p-Determinants and monodromy of differential operators

We review interrelations between arithmetic properties (so-called p-determinants) and analytic properties (eigenvalues of monodromy operators) for differential operators of certain type. This is a joint project with Maxim Kontsevich.

BARBARA PRINARI, University at Buffalo
Breather interactions in the integrable discrete Manakov system

In this talk we will consider a vector generalization of the Ablowitz-Ladik model referred to as the integrable discrete Manakov system. In the focusing regime, this system admits a variety of discrete vector soliton solutions, referred to as fundamental solitons, fundamental breathers, and composite breathers. We will give a full characterization of the interactions of these solitons and breathers, including the explicit forms of their polarization vectors before and after the interaction. Additionally, the results will be interpreted in terms of a Yang-Baxter refactorization property for the transmission coefficients associated with the interacting solitons.

ARCHISHMAN SAHA, University of Ottawa
Deterministic Behaviour in Stochastic Collective Hamiltonian Systems

We consider stochastic perturbations of Hamiltonian systems by noise arising from collective Hamiltonians. We show that these perturbations typically preserve many symmetry-related features of the deterministic system even though the stochastic differential equations governing the dynamics are not symmetric in general. When the deterministic Hamiltonian is symmetric under a free, proper and canonical Lie group action, we show that the projection of a solution of the stochastic system onto the reduced space evolves deterministically. This is joint work with Tanya Schmah (University of Ottawa) and Cristina Stoica (Wilfrid Laurier University).

ALIREZA SHARIFI, University of Manitoba
Integrability and KAM Non–Ergodicity in the Thermostated Hamiltonian Systems

In this talk I will discuss Hamiltonian systems that are thermostated using the Jellinek–Berry (JB) thermostat (J. Phys. Chem. 1988; Phys. Rev. A 1988). Jellinek and Berry proposed this model as a functional extension of Nosé's thermostat (J. Chem. Phys. 1984), introducing several functional parameters that generalize the coupling between the physical system and the thermal reservoir. In molecular dynamics, the JB family aims to generate the canonical ensemble of a Hamiltonian H by coupling H to a one–dimensional heat reservoir with potential energy $v(s)$ and kinetic energy $\frac{1}{2Q}(p_s/u(s))^2$; i.e.,

$$G(x,s,p_s) := \underbrace{H(a(s)\cdot x)}_{\text{Physical system}} + \underbrace{\frac{p_s^2}{2Q\,u(s)^2} + gkT\,v(s)}_{\text{Thermostat}}.$$

I will describe when the JB–thermostated periodic ideal gas is Liouville completely integrable and satisfies a KAM twist condition known as Rüssmann non–degeneracy. This property ensures that the system admits action–angle variables and a nondegenerate frequency map. Using these results, one can show that a thermostated, collisionless, non–ideal gas—that is, a smooth perturbation of the ideal case—possesses a positive–measure set of invariant tori at sufficiently high reservoir temperatures. Consequently, the thermostated dynamics remain non–ergodic in this regime.

The talk will emphasize the geometric structure underlying these results, including the role of symplectic transformations, the existence and persistence of invariant tori, highlighting the connection between thermostat design and classical problems of integrability and ergodicity in Hamiltonian systems.

JACEK SZMIGIELSKI, University of Saskatchewan
Spinor Camassa-Holm Equations

I will outline a construction of spinor analogs of the Camassa-Holm equation. In essence, to each orthogonal group, one can associate a Camassa-Holm-type equation that has a complex internal structure. I will motivate this generalization using the example of spectral deformations of the Euler-Bernoulli beam problem, which corresponds to the Clifford algebra on two generators with Minkowski signature. The talk is based on recent work with R. Beals and ongoing research with A. Hone and V. Novikov.