Probability and PDEs
Probabilité et EDP

(Org: **Vincent Martinez** (CUNY Hunter College), **Geordie Richards** (University of Guelph) and/et **Philippe Sosoe** (Cornell University))

MUSTAFA AVCI, Athabasca University

A viscosity solution approach to the Feynman-Kac formula for a one-dimensional parabolic PDE with variable exponent coefficient

This work establishes the existence and uniqueness of a solution for a one-dimensional parabolic Cauchy problem set on the positive half-line involving coefficients with variable exponent whose generator is associated with a stochastic differential equation involving state-dependent variable exponent. The problem is analyzed within the framework of viscosity solutions, addressing cases where classical solutions may not exist due to insufficient coefficient regularity. We demonstrate that the unique viscosity solution is given by the Feynman-Kac formula, thereby establishing a rigorous link between the probabilistic representation and the analytical solution. A key element of the proof relies on the property that the associated stochastic process remains strictly positive on its state space $(0, \infty)$, which allows for the application of local ellipticity arguments despite potential degeneracy at the boundary. The analysis is completed by applying the standard parabolic regularity theory to show that the viscosity solution possesses local Sobolev regularity in $W^{2,1}_{m,loc}$.

Keywords. stochastic process; viscosity solutions; Feynman-Kac formula; degenerate parabolic PDE; the comparison principle; the dynamic programming principle; local ellipticity.

YURI BAKHTIN, Courant Institute, NYU

Differentiability of the effective Lagrangian for HJB equations in dynamic random environments

We prove differentiability of the effective Lagrangian for continuous space-time multidimensional directed variational problems in random dynamic environments with positive dependence range in space and time. Thus the limiting fundamental solutions in the associated homogenization problems for HJB equations are classical. For several continuous models of FPP and LPP type, our method provides differentiability of limit shapes and shape functions. This is joint work with Douglas Dow.

RALUCA BALAN, University of Ottawa

Recent advances for SPDEs with Lévy noise

In this talk, we introduce a new class of processes that can be used as noise for stochastic partial differential equations (SPDEs). This noise is called the "Lévy colored noise", and is constructed from a Lévy white noise using the convolution with a suitable spatial kernel. We assume that the Lévy measure of the noise has finite variance. The stochastic integral with respect to this noise is constructed similarly to the integral with respect to the spatially-homogeneous Gaussian case considered in Dalang (1999). Using Rosenthal's inequality, we provide an upper bound for the p-th moment of the stochastic integral with respect to this noise. Then, we analyze the existence of moments for linear and non-linear SPDE with this noise, considering as examples the stochastic heat and wave equations. This talk is based on joint work with Juan Jiménez.

BJOERN BRINGMANN, Princeton University

Global well-posedness of the stochastic Abelian-Higgs equations in two dimensions

There has been much recent progress on the local solution theory for geometric singular SPDEs. However, the global theory is still largely open. In this talk, we discuss the global well-posedness of the stochastic Abelian-Higgs model in two dimension, which is a geometric singular SPDE arising from gauge theory. This is joint work with S. Cao.

FRANCESCO CELLAROSI, Queen's University
Stochastic Calculus for the Theta Process

A key step in stochastic analysis is the "art" of giving meaning to integrals against a random function. This is not a trivial task, since interesting random functions have poor regularity and Riemann-type approaches typically do not work. Sometimes, we may exploit the martingale-like properties of our random functions to give meaning to integrals against them. Alternatively, we may employ Rough Path Theory to define stochastic integration, trading some of the nice probabilistic properties for analytical and algebraic ones. I will outline how this is done in the case of the Theta Process. This is a stochastic process of number-theoretical origin that shares several (but not all!) properties with the Brownian Motion, and classical probabilistic tools to define a stochastic calculus for this process cannot be used. Joint work with Zachary Selk (https://arxiv.org/abs/2406.05523)

YU-TING CHEN, University of Victoria
Martingale description of the two-dimensional stochastic heat equation

The two-dimensional stochastic heat equation has a simple, but purely formal, description as the two-dimensional heat equation with a random potential given by space-time white noise. Since this equation poses difficulties for solution theories of stochastic partial differential equations (SPDEs), some approximate solutions developed in the late 90s have played a vital role in subsequent studies. This talk will provide an overview of a precise martingale description of the limiting solutions. The description extends the existing random-field and semigroup formulations of SPDEs. Certain extremal properties of Gaussians, previously known in solvable models of quantum mechanics, are now incorporated.

DUNCAN DAUVERGNE, University of Toronto
Characterization of the directed landscape from the KPZ fixed point

The KPZ universality class is a collection of two-dimensional random metrics and one-dimensional random growth models that exhibit the same universal behaviour under rescaling. The directed landscape is the scaling limit of random metrics in this class. The KPZ fixed point is the scaling limit of random growth models in this class, and arises as a marginal of the directed landscape. In this talk, we will give a characterization of the directed landscape from its KPZ fixed point marginals. For a large range of models, this reduces the problem of proving convergence to the directed landscape to proving convergence to the KPZ fixed point. Joint work with Lingfu Zhang.

NATHAN GLATT-HOLTZ, Indiana University
On Long Time Accuracy for Stochastic Partial Differential Equations Under Approximation.

We are interested in the question of the long time accuracy of solutions of nonlinear stochastic partial differential equations (SPDEs) under various approximations. We will present a powerful and quite general framework which addresses such questions. It centers on 1) proving a (parameter uniform) contraction of the Markovian dynamic in an appropriate Wasserstein metric 2) establishing the accuracy of approximations over a finite time window. Our main paradigmatic application for this approach concerns the numerical approximation of the stochastically driven Navier-stokes equations where the question of long time accuracy was open.

FAUZIA JABEEN, Toronto Metropolitan University
Efficient Method of Estimating Second-Order Sensitivities for Stochastic Discrete Biochemical Systems

Biochemical systems involving some small molecular populations may exhibit stochastic fluctuations that can influence cellular dynamics, thus discrete stochastic models are essential to accurately represent them. Additionally, their models are often stiff due to reactions occurring on widely separated time-scales. Sensitivity analysis is crucial for understanding how parameter changes affect system dynamics and second-order sensitivities provide concavity information and capture interactions between parameters. We propose a finite-difference technique for estimating second-order parametric sensitivities in moderately stiff to stiff stochastic discrete models of biochemical systems. This method uses an adaptive tau-leaping scheme combined with a coupling strategy for nominal and perturbed processes, to achieve both accuracy and computational efficiency. Moreover, this approach may be extended to models of reaction-diffusion systems. This is joint work with Silvana Ilie.

CHRISTOPHER KENNEDY, Queen's University
On the analysis of fluid-solid interactions

I will present a Saint-Venant system of equations for laminar shallow water, which describes the interaction of a viscous and incompressible fluid interacting with a floating rigid body. The governing fluid equations resemble the compressible Navier-Stokes equations with a particular choice of equation of state. Following previous work established by Feireisl on the existence of global-in-time weak solutions in the presence of a non-degenerate viscosity term, I will discuss the general approach of passing to the limit of Faedo-Galerkin solutions of artificially regularised equations using a compactness argument. In particular, following a careful analysis similar to Vasseur and Yu, I will provide an overview of our proof of the existence of weak solutions. I will explain how a jump discontinuity in the height of the fluid domain, when interacting with a partially immersed floating solid, presents an obstacle in proving the global existence of weak solutions. This talk is on joint work with Giusy Mazzone.

ARJUN KRISHNAN, University of Rochester
Field induced phase transition in the polymer model

In three spatial dimensions or higher, it is a classical fact that the directed polymer model has two phases: Brownian behavior at high temperature, and non-Brownian behavior at low temperature. We consider the response of the polymer to an external field or tilt, and show that at fixed temperature, the polymer has Brownian behavior for some fields and non-Brownian behavior for others. In other words, the external field can induce the phase transition in the polymer model.

ZAIB UN NISA MEMON, Toronto Metropolitan University
A hybrid method for stochastic simulations of reaction-diffusion epidemic models

Reactive Multiparticle Collision (RMPC) Dynamics, a particle-based method, is able to keep track of every single individual in a population. However, tracking of infectious individuals becomes infeasible as the cases increase, in which case a compartment-based method, such as Inhomogeneous Stochastic Simulation Algorithm (ISSA), is typically used. This motivated the development of a temporally coupled RMPC-ISSA framework. The hybrid method results in significant acceleration of the simulations of reaction-diffusion epidemic models compared to RMPC-only simulations. This is joint work with K. Rohlf.

MIHAI NICA, University of Guelph
A probabilist's guide to the Hermite polynomials

The Hermite polynomials are intimately connected to Dyson's Brownian motion and other important stochastic processes. In this talk, I will showcase a Gaussian expectation formula for the Hermite polynomials that allows one to easily derive limit theorems for some of these processes and other useful results. Based on this joint work with Janosch Ortmann https://arxiv.org/abs/2508.13910

JEREMY QUASTEL, University of Toronto
Integrable PDE in random growth

We'll survey the appearance of integrable equations such as KP, Toda and Hirota in integrable random growth models.