Number Theory by Early Career Researchers
Théorie des nombres par des chercheurs en début de carrière
(Org: **Jérémy Champagne**, **AJ Fong** and/et **Zhenchao Ge** (University of Waterloo))

ALI ALSETRI, University of Kentucky
Burgess-type character sum estimates over generalized arithmetic progressions of rank 2.

We extend the classical Burgess estimates to character sums over proper generalized arithmetic progressions (GAPs) of rank 2 in prime fields. The core of our proof is a sharp upper bound for the multiplicative energy of these sets, established by adapting an argument of Konyagin and leveraging tools from the geometry of numbers. This is joint work with Xuancheng Shao.

FÉLIX BARIL BOUDREAU, CICMA & Université du Luxembourg
Abelian varieties with homotheties

Let A be an Abelian variety defined over a number field K. The celebrated Bogomolov–Serre theorem states that, for any prime ℓ, the image of the ℓ-adic representation of the absolute Galois group of K contains all c-th power homotheties, where c is a positive integer. When K is a global function field, Zarhin has shown that the corresponding statement fails in general. In this talk, I will present an analogue to Bogomolov–Serre's theorem when K is a finitely generated field of positive characteristic. This is part of an ongoing joint work with Sebastian Petersen (University of Kassel).

HYMN CHAN, University of Toronto
The p-adic Langlands Program and Breuil's Lattice Conjecture

Roughly speaking, the local Langlands correspondence is between n-dimensional Galois representations of $\mathrm{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$ and certain admissible smooth representations of $\mathrm{GL_n}(\mathbb{Q}_p)$. However, if we take the coefficient field to be an extension of the p-adic numbers, the objects on both sides are much more complicated, and it is the study of the p-adic Langlands Program. Currently, it is known only for the group $\mathrm{GL}_2(\mathbb{Q}_p)$. Breuil conjectured a lattice conjecture, which provides evidence for such a correspondence in the case of $\mathrm{GL}_2(K)$, where K is an unramified extension of $\mathbb{Q}_p$.

ALEX COWAN, University of Waterloo
Murmurations from functional equations

Unexpected and striking oscillations in the average a_p values of sets of elliptic curves, dubbed murmurations, were recently discovered using techniques from data science. Since then, similar patterns have been discovered for many other types of arithmetic objects. In this talk we present a new approach for studying murmurations, revolving around mean values of L-functions in the critical strip and guided by random matrix theory. With our approach, we prove murmurations in many cases conditional on standard conjectures, and unconditionally for all GL1 automorphic representations. To handle the case of elliptic curves knowledge is needed of the distribution of conductors of elliptic curves when ordered by height, which is of independent interest.

JOSE CRUZ, University of Calgary
A tale on trascendence and arithmetic equivalence

This talk is about current work in progress that is inspired by TeamsUp! project (25frg504). It is well known that the Dedekind zeta function determines the product of the class number and the regulator of a number field, but not each of these invariants individually. As Perlis showed in the 1970s, there exist number fields with the same Dedekind zeta function but with different regulators and class numbers.

In this talk, we explore a converse phenomenon. Our main result is that, assuming the algebraic independence of logarithms (the weak Schanuel conjecture), two totally real number fields with the same regulator must have the same Dedekind zeta function. As a further consequence of our methods, we show that the weak Schanuel's conjecture implies that the residues at $s = 1$ of two distinct Dedekind zeta functions of totally real number fields are linearly independent over the algebraic closure of $\mathbb{Q}$ in $\mathbb{C}$.

It is worth mentioning that Gun, Murty, and Rath previously proved that the weak Schanuel conjecture implies the transcendence of these residues for arbitrary number fields, so this may be viewed as a very tiny addendum to that story.

NIC FELLINI, Queen's University
Non-Wieferich Primes in Number Fields

An odd prime p is called a base-a Wieferich prime for some integer $a \neq 0, \pm 1$ if

$$a^{p-1} \equiv 1 \bmod p^2.$$

The interest in Wieferich primes stems from their connection to Fermat's Last Theorem. While results on base-a Wieferich primes remain elusive, there has been some progress in understanding their complement, known as non-Wieferich primes, under various hypotheses. In this talk, I will discuss joint work with M. Ram Murty on number field analogues of non-Wieferich primes.

KEIRA GUNN, Mount Royal University
Some Results in Dynamics of the Positive Characterstic Tori

The positive characteristic tori T_F are a set of counterparts to the real torus $T = R/Z$. In positive characteristic we define the "integers" as polynomials with coefficients from a finite field F ($Z_F := F[t]$) and the "reals" as the field of Laurent series with coefficients in F ($R_F := F((t))$) so that the positive characteristic torus over F is similarly defined: $T_F := R_F/Z_F$. While T and T_F have some structural and operational similarities, they behave fundamentally differently, particularly with regards to dynamics. In particular, we find that Furstenberg's orbital density theorem falls apart in positive characteristic, and establish that the intersection of orbits of affine maps rely on sets that depend on powers of the characteristic of F rather than arithmetic progressions. At first glance, the simplicity of working in T_F and its similarities to T suggest that we should be able to find many of the same simple results; however in reality the structure of T_F consists of infinitely defined sub-structures constructed by shifts of Frobenius maps into itself and these sub-structures present themselves frequently in a manner that does not occur in T.

HAZEM HASSAN, McGill University
p-adic higher Green's functions

Heegner cycles are higher weight analogues of Heegner points. Their arithmetic intersection numbers also appear as Fourier coefficients of modular forms and often belong to abelian extensions of imaginary-quadratic fields. The archemdiean contribution to height pairings of Heegner cycles is related to so-called Higher Green's functions, whose algebraicity was conjectured by Gross and Zagier and has been recently proven by Bruinier, Li and Yang.

I will discuss higher weight generalization of Darmon–Vonk's theory of rigid meromorphic cocycles and of Manin–Drinfeld's theta functions. This leads to a definition of p-adic higher Green's function for Heegner cycles on Shimura curves, as well as their conjectured real-quadratic analogue, Stark–Heegner cycles on modular curves. These p-adic functions computationally seem to be producing algebraic numbers in abelian extensions of quadratic number fields, and might be realized as a local contribution to a p-adic height pairing of cycles.

FATEMEH JALALVAND, University of Calgary
Shape of log-unit lattices in D_6 fields

Lattices play an important role in modern mathematics and are essential in Minkowski's geometry of numbers, the sphere-packing problem in Euclidean space, and arithmetic statistics. The shape of a lattice is defined to be its equivalence class up to isometries and homotheties. In number theory, an important class of lattices are log-unit lattices, and a deeper understanding of them leads to new perspectives on computational problems in number fields. In this talk, I will describe the shapes and geometry of log-unit lattices arising from D_6 sextic number fields of signature $(0, 3)$. This work is a result of the TeamsUp! project (25frg504), available at: https://www.birs.ca/events/2025/focussed-research-groups/25frg504.

NICOL LEONG, University of Lethbridge
On some results involving the Riemann zeta function and the Mobius function

The summatory Mobius function M(x) is closely related to the notorious Riemann zeta function and is an object of much study. It is well known that while Merten's conjecture is false, proving something just slightly weaker would imply Riemann's hypothesis. In this talk, we present some recent unconditional explicit results on the summatory Mobius function M(x) (joint with Ethan Lee), while along the way proving some improvements in lower bounds for the zeta function. Finally we will discuss some ongoing joint work with Nathan Ng on the logarithmic density of the set of reals for which Merten's conjecture fails.

ISABELLA NEGRINI, University of Toronto
Rigid Cocycles and the p-adic Kudla Program

Rigid cocycles, introduced by Darmon and Vonk in 2017, offer a promising framework to extend complex multiplication theory to real quadratic fields, suggesting a theory of "real multiplication." They exhibit striking parallels with modular forms and are central to the emerging p-adic Kudla program. While the classical Kudla program studies the theta correspondence between automorphic forms on different groups, the p-adic version appears to replace automorphic forms with rigid cocycles. Although a theory for a p-adic theta correspondence has yet to be developed, recent results suggest its existence. In this talk, I present some of these p-adic results, draw comparisons to the classical setting, and discuss the evidence for an underlying p-adic theta correspondence.

PAUL PÉRINGUEY,
Joint distributions of error terms for primes in arithmetic progressions modulo 11

In 1983, Bays and Hudson noted two surprising phenomena for the prime race mod 11. First, that the leading residue class seems to cycle through a pattern with only minor deviations; and secondly, that the trailing residue class tends to be the additive inverse of the leading one.

In this talk, I will discuss, assuming GRH and the Linear Independence of zeros of Dirichlet L-functions, a formula for the logarithmic density of the set of positive real numbers on which two prime counting functions $\psi(x; q, a)$ and $\psi(x; q, b)$ are simultaneously larger than their asymptotic main terms. This formula provides, when $q = 11$, a deeper mathematical explanation of the phenomena observed by Bays and Hudson. This is joint work with Kübra Benli and Greg Martin.

EMILY QUESADA-HERRERA, University of Lethbridge
On the vertical distribution of the zeros of the Riemann zeta-function

In 1973, assuming the Riemann hypothesis (RH), Montgomery studied the vertical distribution of zeta zeros, and conjectured that they behave like the eigenvalues of some random matrices. We will discuss some models for zeta zeros – starting from the random matrix model but going beyond it – and related questions, conjectures and results on statistical information on the zeros. In particular, assuming RH and a conjecture of Chan for how often gaps between zeros can be close to a fixed non-zero value, we will discuss our proof of a conjecture of Berry (1988) for the number variance of zeta zeros, in a regime where random matrix models alone do not accurately predict the actual behavior (based on joint work with Meghann Moriah Lugar and Micah B. Milinovich).

FATEME SAJADI, University of Toronto
A Unified Finiteness Theorem For Curves

This talk presents a unified framework for finiteness results concerning arithmetic points on algebraic curves, exploring the analogy between number fields and function fields. The number field setting, joint work with F. Janbazi, generalizes and extends classical results of Birch–Merriman, Siegel, and Faltings. We prove that the set of Galois-conjugate points on a smooth projective curve with good reduction outside a fixed finite set of places is finite, when considered up to the action of the automorphism group of a proper integral model. Motivated by this, we consider the function field analogue, involving a smooth and proper family of curves over an affine curve defined over a finite field. In this setting, we show that for a fixed degree, there are only finitely many étale relative divisors over the base, up to the action of the family's automorphism group (and including the Frobenius in the isotrivial case). Together, these results illustrate both the parallels and distinctions between the two arithmetic settings, contributing to a broader unifying perspective on finiteness.

GIAN CORDANA SANJAYA, University of Waterloo
Squarefree density of discriminant of polynomials with restricted coefficients

The squarefree density problem, which asks to determine the probability that a multivariate integer polynomial $F(x_1, \dots, x_n)$ attains a squarefree value, is a classical problem. Some recent progress has been made in the case where F is the discriminant of a monic integer polynomial. Namely, Bhargava, Shankar, and Wang proved that the density of monic integer polynomials with squarefree discriminant exists and is given by the product of the local densities, which were previously computed by Yamamura.

In this talk, I will discuss the case where F is the discriminant of a monic integer polynomial with restricted coefficients, with the emphasis on local density computations. This talk is partially based on a joint work with Valentio Iverson and Xiaoheng Wang.

ANTON SHAKOV, Queen's University
Some Distributional Properties of 2-Regular Integer Sequences

The class of k-regular sequences provides an important generalization of automatic sequences and is deeply connected to many topics in number theory. We generalize a recent result of Bettin, Drappeau, and Spiegelhofer on the statistical distribution of Stern's diatomic sequence to show that a large family of 2-regular integer sequences obey a log-normal statistical distribution. Our approach relies on viewing the distribution of k-regular sequences from the perspective of products of random matrices. We discuss a connection between a specific 2-regular integer sequence and the arithmetic function $\tau(n^2+1)$.

KYLE YIP, Georgia Institute of Technology
Diophantine tuples and Diophantine powersets

Let k, n be integers with $k \geq 2$ and $n \neq 0$. A set A of positive integers is a Diophantine tuple with property $D_k(n)$ if the product of $ab+n$ is a perfect k-th power for every $a, b \in A$ with $a \neq b$. These Diophantine tuples have been studied extensively. In this talk, I will discuss some recent progress on "Diophantine powersets" (first studied by Gyarmati, Sárközy, and Stewart), where we allow $ab+n$ to be a perfect power instead of a perfect k-th power for some fixed k. Joint work with Ernie Croot.

XIAO ZHONG, University of Waterloo
A dynamical Manin–Mumford type question on polynomial endomorphisms of $\mathbb{A}^2$

In this talk, I will discuss regular polynomial endomorphisms of $\mathbb{A}^2$ that admit infinitely many periodic curves. I will show that if a family of curves contains a Zariski dense set of periodic curves under a regular polynomial endomorphism F, then this family is invariant under some iterate of F. Furthermore, I will present a complete classification of regular polynomial endomorphisms that have infinitely many periodic curves of bounded degree. As a consequence, we obtain a weaker form of a special case of the Dynamical Manin–Mumford Conjecture, a dynamical analogue of the classical Manin–Mumford Conjecture in arithmetic geometry.