**New trends in Analysis**
**Nouvelles tendances en matière d'analyse**
(Org: **Almut Buchard** (University of Toronto) and/et **Angel Martinez** (CUNEF Universidad, Madrid))

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**FRANCISCO TORRES DE LIZAUR**, Universidad de Sevilla
*Symmetries of eigenfunctions*

The symmetry conjecture asks whether, on a closed Riemannian manifold, the signs of laplace eigenfunction in the $\lambda \to \infty$ limit are evenly distributed. In the talk I will report on recent progress on this question and related problems on the properties of the value distribution of eigenfunctions. This is joint work with Ángel David Martínez.

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**ROBERT HASLHOFER**, University of Toronto
*Free boundary minimal disks in convex balls*

The classical Lusternik-Schnirelman theorem says that any 2-sphere equipped with an arbitrary metric contains at least 3 embedded geodesic loops. Moving up one dimension, Yau asked about the existence of multiple embedded minimal surfaces of simple topology, namely minimal 2-spheres in 3-spheres or minimal 2-disks in 3-balls. In this talk, I will discuss joint work with Dan Ketover, where we show that every strictly convex 3-ball with nonnegative Ricci-curvature contains at least 3 embedded free boundary minimal 2-disks for any generic metric, and at least 2 solutions even without genericity assumption. Our approach combines ideas from min-max theory, geometric flows, and degree theory.

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**DMITRY JAKOBSON**, McGill
*Nodal solutions of Yamabe equations and curvature prescription*

We discuss several old and new results about conformal invariants arising from nodal solutions of Yamabe type equations, and applications to curvature prescription

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**DAN MANGOUBI**, Hebrew University
*On common roots of Legendre polynomials*

Let $S$ be a closed Riemannian surface, and $\gamma \subset S$ be a curve. An old question in Spectral Geometry asks how many Laplace eigenfunctions can vanish on $\gamma$. Bourgain and Rudnick showed that on the flat torus $\mathbb{T}^2$ this number is finite unless $\gamma$ is a closed geodesic. A conjecture by Stieltjes says that no two zonal spherical harmonics may vanish on the same small circle, or in arithmetic terms, no two Legendre polynomials share a non-zero common root. We show that the number of Legendre polynomials which share any given non-zero root is finite. The talk is based on joint work in progress with Borys Kadets.

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**ALBA DOLORES GARCÍA RUIZ**, CUNEF Universidad
*High-Energy Laplace Eigenfunctions on Integrable Billiards*

A famous conjecture by Berry suggests that, in chaotic dynamical systems, Laplace eigenfunctions, with specific boundary conditions, resemble to random monochromatic waves; however, this behavior is generally not expected in integrable dynamical systems. Here, we explore the behavior of high-energy eigenfunctions and their connection to Berry's random wave model. In particular, we study a related property, which we call Inverse Localization, describing how eigenfunctions can approximate monochromatic waves in small regions of the domain.

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**BRUNO STAFFA**, Rice University
*Density and equidistribution of closed geodesics and stationary geodesic nets*

This talk will be about the distribution of closed geodesics and stationary geodesic nets in a closed Riemannian manifold $(M, g)$. When $\dim(M) = 2$, together with Xinze Li we could prove that for a generic metric $g$, there exists an equidistributed sequence of closed geodesics in $(M, g)$. When $\dim(M) \geq 3$, in collaboration with Yevgeny Liokumovich we showed that stationary geodesic nets (which are analogs of closed geodesics whose domain is a graph instead of a circle) are dense. In fact, one can obtain generic equidistribution of these objects. The main tools used to prove these results were the Almgren Pitts Min-Max Theory (in particular the Weyl Law for the Volume Spectrum) and a Structure Theory for stationary geodesic nets analogous to that of Brian White for minimal submanifolds.

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**JOHN TOTH**, McGill University

*$L^2$ restriction bounds for analytic continuations of quantum ergodic Laplace eigenfunctions.*

We prove a quantum ergodic restriction (QER) theorem for real hypersurfaces $\Sigma \subset X$, where $X$ is the Grauert tube associated with a real-analytic, compact Riemannian manifold. As an application, we obtain $h$ independent upper and lower bounds for the $L^2$ - restrictions of the FBI transform of quantum ergodic Laplace eigenfunctions restricted to $\Sigma$ satisfying certain generic geometric conditions. This is joint work with X. Xiao.

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**JÉRÔME VETOIS**, McGill University

*Nonexistence of extremals for the second conformal eigenvalue in low dimensions*

In this talk, we will consider the second conformal eigenvalue on a closed Riemannian manifold of positive Yamabe type and dimension greater than or equal to 3. The second conformal eigenvalue is defined as the infimum of the second eigenvalue of the conformal Laplacian in a conformal class of metrics with renormalized volume. We will discuss a recent result showing that this infimum is not attained for metrics close to the round metric on the sphere in dimensions 3 to 10, which contrasts sharply with the situation in dimensions greater than or equal to 11, where Ammann and Humbert obtained the existence of minimizers on any closed nonlocally conformally flat manifold. This is a joint work with Bruno Premoselli (Université Libre de Bruxelles).