Mathematical Relativity and Geometric Analysis
Relativité mathématique et analyse géométrique
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AMIR BABAK AAZAMI, Clark University
Normal forms, "almost-Einstein" metrics, and conformal invariants

A semi-Riemannian manifold has a "normal form" if its curvature tensor is determined by just the critical points and values of its sectional curvature. Examples include Riemannian Einstein 4-manifolds and the classification of Lorentzian spacetimes by their Petrov Types. In this talk we will combine these two cases, yielding new examples of distinguished "almost-Einstein" metrics. We will also briefly discuss higher-dimensional analogues of these results, focusing on conformal invariants.

TRACEY BALEHOWSKY, University of Calgary
The Inverse Problem of Recovering a Riemannian Metric from Area Data

Broadly speaking, there are two classes of inverse problems — those that are concerned with the analysis of PDEs, and those that are geometric in nature. In this talk, I will introduce the audience to these classes by highlighting classical examples. Then, I will introduce the geometric problem of recovering a Riemannian metric from area data. I will connect this problem to questions which arise in the AdS/CFT correspondence. I will survey several cases where it is possible to determine a metric from knowledge of the areas of certain families of surfaces. A key feature of these results is that they combine techniques from both PDE and geometric perspectives.

IVAN BOOTH, Memorial University
Black hole evolution and internal structure: constraints from the stability operator

For over 50 years apparent horizons have been the canonical examples of marginally outer trapped surfaces (MOTS). However, in the last few years it has come to be understood that they are not the only examples. In fact, most MOTS are not horizons but instead lurk deep inside black holes. Not only can they be used to better understand black hole internal structure but also these internal MOTS play key roles in dynamical events such as black hole mergers. Geometrically MOTS are close relatives of minimal surfaces from Riemannian geometry and the spectra of their analogous stability operators play a key role in determining their behaviour and properties. In this talk I will review some recent work with G.Cox and C.M.Okpala in which we define a generalized stability operator, study its properties, and then use analytical and numerical techniques to explore the implications for black hole geometry and evolution.

GRAHAM COX, Memorial University of Newfoundland
Black hole mergers and bifurcations of marginally outer trapped surfaces

It is well known that a marginally outer trapped surface evolves smoothly in time if 0 is not an eigenvalue of its stability operator; otherwise it may bifurcate. In this talk I will present geometric criteria for saddle-node, transcritical and pitchfork bifurcations to occur as the initial data is varied. These results apply to any variation of the initial data, not just time evolution. Using these criteria, I will show the existence of an infinite sequence of pitchfork and transcritical bifurcations along the inner horizon in the Reissner-Nordström spacetime, as the charge varies between zero and the extremal value. This talk represents joint work with Liam Bussey and Hari Kunduri.

NISHANTH GUDAPATI, College of the Holy Cross
Remarks on the s=1 Teukolsky Equation

The mathematical problem of stability of Kerr black hole spacetimes has been a major subject in mathematical and theoretical studies of the Einstein equations of general relativity. In the 1970s, a major breakthrough was achieved when Teukolsky was able to construct a master equation for the gauge-invariant ('extreme') Newman-Penrose scalars.

The structure of the linearized Einstein equations is such that the Teukolsky master equation does not admit a natural variational formulation for higher spin fields, such as the Maxwell and the linearized Einstein fields. As a result the energy methods are not directly applicable for this equation.

In this talk, we shall present the construction of a positive-definite energy for the Teukolsky variables for the Maxwell fields (spin s=1) in the special case of axial symmetry. The origin of this positive-definite energy is a Hamiltonian principle and the construction is based on using certain 'twist' potentials as the main variables as opposed to the vector potential.

JEFF JAUREGUI, Union College (NY)
Optimizing capacity with nonnegative scalar curvature

Inspired by Bartnik's well-known problem of minimizing the ADM mass among all asymptotically flat manifolds of nonnegative scalar curvature extending some compact region, we will discuss a complementary problem of maximizing the capacity over the same class of objects. We will also tie this in with joint work with Raquel Perales and Jim Portegies on the upper semicontinuity of capacity for Sormani–Wenger intrinsic flat convergence.

NIKY KAMRAN,
Global counterexamples to uniqueness for a Calderón problem with C^k conductivities

Let Ω be a bounded subset of $\mathbb{R}^n$ with C^∞ boundary, where $n \geq 3$, and let $\gamma = (\gamma^{ij})$ be a bounded measurable function on $\overline{\Omega}$ taking values in the set $\mathcal{S}_n$ of positive definite $n \times n$ symmetric matrices. The Calderón inverse problem consists in recovering the map γ from the from the knowledge of the Dirichlet-to-Neumann map at fixed frequency for the operator $L_\gamma = -\partial_i(\gamma^{ij}\partial_j)$, up to some gauge equivalences induced by the invariance properties of the Dirichlet-to-Neumann map. We obtain counterexamples to uniqueness for the Calderón problem by showing that for any smooth map γ and any frequency that does not belong to the Dirichlet spectrum of L_γ, there exists, for any $k \geq 1$ and any $\epsilon > 0$, a pair of non gauge-equivalent maps γ_1, γ_2 of class C^k which are ϵ-close to γ in the $C^k(\overline{\Omega}, \mathcal{S}_n)$ topology, such that their Dirichlet-to-Neumann maps are equal. This is joint work with Thierry Daudé (Besançon), Bernard Helffer (Nantes) and François Nicoleau (Nantes).

MARIEM MAGDY, Perimeter Institute for Theoretical Physics
Estimates for spinor fields using the space-spinor formalism

I will discuss ongoing work on estimates for spinor fields obeying first-order equations, formulated within the space–spinor framework. The approach is motivated by the positive commutator method presented in the work of P. Hintz and A. Vasy, originally designed for tensor fields satisfying second-order equations. In this talk, I will outline how these ideas can be adapted to the spinorial setting and how the resulting estimates fit naturally into the first-order structure of the equations.

ARGAM OHANYAN, University of Toronto
On the geometry of continuously differentiable spacetime metrics

Nonsmooth metrics appear naturally in mathematical relativity and spacetime geometry, whether as solutions of Einstein's equations or during the course of physically relevant operations such as spacetime matching. It is important to study their geometry also to give further physical credence to the singularity theorems of Hawking and Penrose. In this talk, we will discuss the class of continuously differentiable spacetime metrics, for which many important theorems have been established recently. After introducing the main approximation tools and methods of study, we will discuss important applications such as the Hawking and Penrose singularity theorems, the Hawking-Penrose singularity theorem, and the splitting theorem.

This talk is partly based on joint collaborations with Kunzinger, Schinnerl, Steinbauer and with Braun, Gigli, McCann, Sämann.

YAKOV SHLAPENTOKH ROTHMAN, University of Toronto
Polynomial Decay for the Klein-Gordon Equation on the Schwarzschild Black Hole

We will start with a review of previous instability results concerning solutions to the Klein-Gordon equation on rotating Kerr black holes and the corresponding conjectural consequences for the dynamics of the Einstein-Klein-Gordon system. Then we will discuss recent work where we show that, despite the presence of stably trapped timelike geodesics on Schwarzschild, solutions to the corresponding Klein-Gordon equation arising from strongly localized initial data nevertheless decay polynomially. Time permitting we will explain how the proof uses, at a crucial step, results from analytic number theory for bounding exponential sums. The talk is based on joint work(s) with Federico Pasqualotto and Maxime Van de Moortel.

RYAN UNGER, University of California, Berkeley
The moduli space of dynamical spherically symmetric black hole spacetimes and the extremal threshold

Extremal black holes are special solutions of Einstein's equations with maximal spin or charge for their mass. In this talk, we consider the moduli space of spherically symmetric solutions to the Einstein-Maxwell equations with a real scalar field. It is known that solutions in this space either form black holes or disperse (no singularity forms). In the small scalar field regime, we show that the interface between these two regions of moduli space is a smooth hypersurface consisting of asymptotically extremal black holes. This is based on upcoming joint work with Yannis Angelopoulos (BIMSA) and Christoph Kehle (MIT).

JAMES WHEELER, University of Michigan
Asymptotically Euclidean Solutions of the Constraint Equations with Prescribed Asymptotics

I will discuss ongoing work on the construction of asymptotically flat vacuum initial data sets in General Relativity via the conformal method. My collaborators (Lydia Bieri, David Garfinkle, Jim Isenberg, and David Maxwell) and I have demonstrated that certain asymptotic structures may be prescribed a priori through the method's seed data, including the ADM momentum components, the leading- and next-to-leading-order decay rates, and anisotropy in the metric's mass term, yielding a recipe to construct initial data sets with desired asymptotics. As an application, we discuss a simple numerical example, with stronger asymptotics than have been presented in previous work, of an initial data set whose evolution does not exhibit the conjectured antipodal symmetry between future and past null infinity.

ERIC WOOLGAR, University of Alberta
Marginally outer trapped surfaces governed by Bakry-Émery Ricci curvature bounds

Recent efforts to define energy conditions synthetically often assume a reference measure on spacetime. In the smooth context, reference measures arise in many applications. In a warped product (Kaluza-Klein) spacetime the reference measure describes the volume of the fibre (internal space), and in conformally invariant formulations of some classical theorems the reference measure provides a conformal scale factor. The corresponding generalization of Ricci curvature is the Bakry-Émery Ricci curvature. Using this approach, I will give a conformally invariant formulation of the Penrose singularity theorem and then explore constraints on horizon topology when the dominant energy condition is replaced by a Bakry-Émery type condition. This can be thought of as the dominant energy condition applied only to "base null directions" in a warped product.

The talk is based on recent work with Eric Ling and Argam Ohanyan.