Logic in Canada IV

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DIEGO BEJARANO, York University
Finding Order in Metric Structures

In continuous logic, there are plenty of examples of interesting stable metric structures. On the other side of the SOP line, there are only a few metric structures where order is relevant, and order appears in a different way in each of them. In this talk, joint work with Aaron Anderson, we present a unified approach to linear orders in continuous logic. We axiomatize these theories, and find generic completions in the ultrametric case, analogous to the complete theory DLO. We study some expansions of these theories, including real closed metric valued fields, from this perspective, and characterize which expansions of metric linear orders should be considered o-minimal.

CHRISTINE EAGLES, University of Waterloo
Algebraic independence of solutions to multiple Lotka-Volterra systems

A major problem in recent applications of the model theory of DCF_0 is determining when a given system of algebraic differential equations defines a strongly minimal set. A definable set S is *strongly minimal* if it is infinite and for any other definable set R (over any set of parameters), either $S \cap R$ or $S \setminus R$ is finite. In joint work with Yutong Duan and Léo Jimenez, we classify exactly when the solution set to a Lotka-Volterra system is strongly minimal. In the strongly minimal case, we classify all algebraic relations between Lotka-Volterra systems and show that for any distinct solutions $x_1, ..., x_n$ (not in the algebraic closure of the base field F), $\mathrm{trdeg}(x_1, \cdots, x_m/F) = 2m$.

ALI HAMAD, University of Ottawa
Bundles of metric structures as left ultrafunctors

The ultraproduct construction play a fundamental role in both classic and continuous first-order logic. Categorical treatment of that construction can be done in the framework of ultracategories first introduced by Makkai and then by Lurie, where it was used in classic model theoretic and topos theoretic settings. We have used this new framework to study categories of models of continuous logic, and showed a result related to bundle theory. A certain class of functors from a compact Hausdorff space to the category of models of a continuous theory is equivalent to a nice enough notion of bundles of models of this theory, with the compact Hausdorff space being the base space. This notion allows for the recovery of familiar notions of bundles like Banach bundles and continuous fields of C^* algebras.

LEO JIMENEZ, The Ohio State University
Pfaffian functions and model theory

Pfaffian functions, which are defined as solutions of triangular systems of order one polynomial differential equations, have played an important role in the model theory of the real field, thanks to their finiteness properties. However, determining whether solutions of a given differential equation are Pfaffian remains a challenging problem. In this talk, I will discuss some work in progress, joint with James Freitag and Ronnie Nagloo, which uses model-theoretic tools to find criteria for functions to be pfaffian.

JOEY LAKERDAS-GAYLE, University of Waterloo
Computability theory of function composition

We will discuss the relationship between function composition and the computability-theoretic complexity of functions (their *partial degree*). We fully characterize the possible degrees of $g \circ f$ in terms of the degrees of f and g. We also consider the problem of "splitting over composition": Given a function h, what are the possible degrees of functions f and g for which $g \circ f = h$? We will discuss some new results, natural examples, and application to computable structure theory.

ILGWON SEO, McMaster University

O-minimality of almost regular multisummable germs

As a step toward addressing Dulac's problem, the main goal of my project is to show the o-minimality of an algebra containing multisummable functions and almost regular germs. A multisummable function can be expressed as a series of holomorphic functions at 0. Together with Patrick Speissegger, I constructed an algebra by replacing holomorphic functions with a.r. germs. However, since the asymptotic expansion of an almost regular germ is generally divergent, this algebra fails to satisfy the closure properties required for o-minimality. To overcome this difficulty, we introduce a refined algebra Q, obtained by selectively choosing well-behaved functions. In this follow-up to my spring talk, I will present what has changed over the summer and explain how the algebra Q can be used to prove o-minimality.

MATHIAS STOUT, McMaster University/Fields Institute

Integration in Hensel minimal fields

I will discuss the construction of Hrushovski-Kazhdan style integral for Hensel minimal fields, generalizing their construction in the V-minimal setting.

This talk will consist of two smaller parts, respectively centered around the following questions

1. What is Hensel minimality and why is it a natural and desirable setting to work in?

2. How to read our main theorem, and why is it (un)surprising?

This is based on joint work with Floris Vermeulen.

SPENCER UNGER, University of Toronto

Circle squaring with algebraic irrationals and few pieces

We show that a closed disk and square with the same area in the plane are equidecomposible using translations whose coordinates are linear combinations of algebraic irrationals. This solves a question of Laczkovich from 1990. Our proof uses a new method for bounding the discrepancy of product sets in the k-torus using the Erdos-Turan inequality. As an application of our work, we obtain an improved upper bound on the number of pieces required to square the circle. For this we fix certain algebraic irrationals and make use of: (1) effective constants in Roth's theorem on diophantine approximation, (2) an idea of Frank Calegari for bounding sums of products of fractional parts of those numbers and (3) computer assistance. This is joint work with Andrew Marks.