Horizons in Operator Algebras
Horizons dans les algèbres d'opérateurs

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JANANAN ARULSEELAN, Iowa State University

Model Theory of von Neumann Algebras: Beyond Tracial States

The majority of the work done to date in the model theory of von Neumann algebras has been done with the assumption of the existence of a faithful normal tracial state. In this setting, model theory has made enough of an impact that one may wonder about other cases, such as type III von Neumann algebras or type II_∞ factors together with their semifinite tracial weight. In these more general settings, one runs into interesting issues involving the Tomita-Takesaki modular theory and left Hilbert algebras. I will survey some of these issues as they relate to two new frameworks for model theory of von Neumann algebras. One of these frameworks is for σ-finite von Neumann algebras and is joint work with Goldbring, Hart, and Sinclair. The other is for von Neumann algebras in full generality. In the latter setting, we encounter a brand new ultraproduct construction. I will discuss this new ultraproduct and the ways model theory helps us to characterize it, following ideas of Ando and Haagerup.

BRANIMIR CACIC, University of New Brunswick, Fredericton

Revisiting the differential topology of higher-dimensional noncommutative tori

The earliest results in noncommutative (NC) differential geometry à la Connes concern the differential topology of higher-dimensional NC tori. In this talk, I'll sketch how these results interface with recent progess in NC differential and Riemannian geometry. In particular, I'll sketch how Elliott's calculation of the diffeomorphism group of a Diophantine-irrational NC 2-torus generalizes to higher dimensions and how Rieffel–Schwarz and Li's classification of higher-dimensional NC tori up to complete Morita equivalence can be refined to classify NC Hermitian line bundles with unitary connection on totally irrational higher-dimensional NC tori up to gauge equivalence. This is partly based on joint work with T. Venkata Karthik.

KEN DAVIDSON, U.Waterloo and U.Ottawa

Large Perturbations of Nest Algebras

Let $\mathcal{M}$ and $\mathcal{N}$ be nests on separable Hilbert space. If the two nest algebras are distance less than 1 ($d(\mathcal{T}(\mathcal{M}),\mathcal{T}(\mathcal{N}))<1$), then the nests are distance less than 1 ($d(\mathcal{M},\mathcal{N})<1$). If the nests are distance less than 1 apart, then the nest algebras are similar, i.e. there is an invertible S such that $S\mathcal{M}=\mathcal{N}$, so that $S\mathcal{T}(\mathcal{M})S^{-1}=\mathcal{T}(\mathcal{N})$. However there are examples of nests closer than 1 for which the nest algebras are distance 1 apart.

ANDREW DEAN, Lakehead University

Classification problems concerning real structures and gradings

We shall give a survey and update on progress for problems concerning classification of real structures and gradings on C^*-algebras, and their associated range of invariant problems.

REMUS FLORICEL, University of Regina

The spectral C^*-algebra of a product system

The spectral C^*-algebra $C^*(E)$ associated with a product system $E=\{E(t)\}_{t>0}$ was introduced by Arveson in the 1990s and serves as a natural "spectrum" for E, linking the representation theory of the product system with that of the algebra itself. In this talk, I will give an overview of the construction and main properties of $C^*(E)$, and then discuss some recent progress on classifying these C^*-algebras.

SAEED GHASEMI, Lakehead University
Preservation of Elementary Equivalence under Tensor Products

The study of how elementary equivalence behaves under various operations on structures traces back to 1959, when Feferman and Vaught proved that it is preserved under reduced products. Although elementary equivalence is not, in general, preserved under tensor products in the category of C*-algebras, it remains open whether this preservation holds in the category of tracial von Neumann algebras. In joint work with Ilijas Farah, we applied a Feferman–Vaught–type theorem to show that tensoring with type I algebras does preserve elementary equivalence within the category of tracial von Neumann algebras.

BRADD HART, McMaster University
Decidedly undecidable results in operator algebras

It is difficult to recall that it has only been 6 years (essentially the last time the CMS meeting was held at the Chelsea Hotel!) since the start of what I would call the era of undecidability in the model theory of operator algebras. These results have followed from the breakthrough results in quantum complexity that goes by the call signal MIP*= RE. I will survey some of the highlights of the past few years, look at the role of embedding problems and say something about current open problems in the area.

CRISTIAN IVANESCU, MacEwan University
Preservation of the Way-Below Relation Under Tensor Products

We show that the way-below relation is preserved under tensor products. While completing this work, we became aware that, in the context of the Cuntz semigroup, an instance of this result was previously established by Antoine, Perera, and Thiel. We nevertheless present our proof here, as it provides a complementary approach and may offer additional insight into the behaviour of the way-below relation under tensorial constructions.

FEODOR KOGAN, University of Toronto
Groupoid models of irrational rotation algebras

From the works of Xin Li we know that every classifiable C^*-algebra has a groupoid model, combining that with the work of Robin Deeley, Ian Putnam, and Karen Strung which provides countably many non-isomorphic groupoid models of the Jiang-Su algebra it follows that every classifiable C^*-algebra also has uncountably many groupoid models, although it is difficult to express these models concretely. In my talk I will show how to build countably many non-isomorphic groupoid models of the irrational rotation algebra by looking at a family of transversals for the Kronecker foliation of the torus.

DAVID KRIBS, University of Guelph
Operator Algebra Perspective on Entanglement-Assisted Quantum Codes

The idea of using entanglement as a resource in quantum computing and communication has been around for a long time. Two decades ago, 'entanglement-assisted' quantum codes were introduced in quantum error correction (EAQEC) as a resource for boosting transmission rates when a sender and receiver share pre-existing entanglement. Shortly thereafter, a pair of (not so clearly related I'd say) generalizations of EAQEC were formulated for 'subsystem codes' and for the classical enhancement of quantum code transmission. As it turns out, each of these three types of code can be viewed as special cases of a general framework for EA codes built on an operator algebra approach to quantum error correction. In addition to unifying these code types under a single umbrella, the resulting framework (EAOAQEC) yields new types of EA codes. In this talk, I'll give a brief introduction to entanglement-assisted codes, the EAOAQEC framework, and some of our results (time dependent). This talk is based on joint work with Serge Adonsou, Guillaume Dauphinais, Priya Nadkarni, and Michael Vasmer.

HUAXIN LIN, Shanghai Institute for Mathematics and Interdisciplinary Sciences
Almost commuting selfadjoint operators and quantum mechanics

We show that Mumford's Approximately Macroscopically Unique (AMU) states exist for quantum systems consisting of unbounded self-adjoint operators when the commutators are small. In particular, AMU states always exist in position and momentum systems when the Planck constant $|\hbar|$ is sufficiently small. However, we show that these standard quantum mechanical systems are far away from classical mechanical (commutative) systems even when $|\hbar| \to 0$.

AAREYAN MANZOOR, University Of Waterloo
There is a non-Connes embeddable Equivalence Relation

Connes embeddability of a group is a finite dimensional approximation property. It turns out this property depends only on the group von Neumann algebra. The property can be extended to all tracial von Neumann algebras. The fact that there is a von Neumann algebra without this property was proved in 2020 using the quantum complexity result MIP*=RE. It is still open for group von Neumann algebras. I will discuss the best-known partial result, which is that there is a group action without this property. In particular, this implies the negation to the Aldous-Lyons conjecture, a big problem in probability theory about finite approximability of a certain class of random graphs.

PATRICK MELANSON, University of Regina
Pro-Tori and Inductive Limits of Non-Commutative Tori

It is a known result that the inductive limit non-commutative tori (also called irrational rotation algebras) can be realized as a non-commutative N-solenoid. Both of these are examples of twisted group C*-algebras. We can generalize to non-commutative d-tori (the higher dimensional analogue) and perform a similar analysis. That is, given a family of non-commutative d_k-tori, $d_k > 1$, and a sufficiently nice embeddings (that is, unital *-homomorphisms), we can realize their inductive limit as a twisted C*-algebra, analogous to a higher dimensional non-commutative N-solenoid.

MEHDI MORADI, University of Ottawa
On locally finite-dimensional traces

Locally finite-dimensional (LFD) traces emerged out of Popa's "local finite-dimensional approximation" as an intrinsic property of C^*-algebras. In my talk, I will provide a new characterization of LFD-traces in terms of projections in the enveloping von Neumann algebra. Then I will address some long-standing questions regarding LFD-traces and their behaviour under convex combinations. I will finish by giving a reformulation of the Universal Coefficient Theorem question in terms of strongly self-absorbing algebras.

ZHUANG NIU, University of Wyoming
$\mathcal{Z}$-absorption and small boundary property

A C*-algebra is said be $\mathcal{Z}$-absorbing if it absorbs the Jiang-Su algebra $\mathcal{Z}$ tensorially. Let us consider a free and minimal $\mathbb{Z}^d$ action on a compact metrizable space, and let us consider the corresponding crossed product C*-algebra. Then it is shown recently that the $\mathcal{Z}$-absorption of the crossed product C*-algebra, which is a C*-algebra property, indeed is completely characterized by the small boundary property (or the zero mean dimension) of the standard Cartan subalgebra, which is a dynamical system property. Along the way, the $\mathcal{Z}$-absorption of the crossed product C*-algebra is also shown to be equivalent to other (C*-algebra or Cantan subalgebra) properties such as uniform property Gamma or zero real rank of certain sequence algebras. The talk is base on joint works with George A. Elliott.

DOLAPO OYETUNBI, University of Windsor
Maximality and symmetry related to the 2-adic ring C-algebra*

The 2-adic ring C*-algebra $\mathcal{Q}_2$ is the universal C*-algebra generated by a unitary and an isometry satisfying certain relations. It contains a canonical copy of the Cuntz algebra $\mathcal{O}_2$, which we denote by the same symbol. In this talk, we explore the maximality of this canonical copy of $\mathcal{O}_2$ inside $\mathcal{Q}_2$, the fixed-point algebra of a periodic *-automorphism σ of $\mathcal{Q}_2$ extending the flip-flop automorphism of $\mathcal{O}_2$, and the relationship between these two notions for $\mathcal{Q}_2$. This is based on joint work with Dilian Yang.

EBRAHIM SAMEI, University of Saskatchewan
Tempered representations on stationary spaces

Let G be a countable discrete group, and let μ be a probability measure on G with finite Shannon entropy. We use ideas from harmonic analysis to generalize Shannon and Avez entropies, taking into account the given weight ω on G, and investigate their relations together as well as to the actions of G on measurable stationary spaces. We apply our approach to show that for a large class of groups (e.g. groups with rapid decay) and probability measures on them, tempered representations on a μ-stationary space (X, ξ) are precisely those satisfying

$$h(G, \mu) = h_\mu(X, \xi),$$

where $h(G, \mu)$ is the Avez entopy of μ and $h_\mu(X, \xi)$ is the Furstenberg entropy of (X, ξ). This implies that these types of stationary spaces are precisely measure-preserving extension of the Poisson boundary of (G, μ). In particular, (X, ξ) is an amenable (G, μ)-space if and only if it is a measure-preserving extension of the Poisson boundary of (G, μ).

This is a join work with Benjamin Anderson-Sackaney, Tim de Laat, and Matthew Wiersma.

CHRISTOPHER SCHAFHAUSER, University of Nebraska - Lincoln
KK-rigidity of simple nuclear C-algebras*

A landmark result in C*-algebra theory is the classification of unital separable simple nuclear Z-stable C*-algebras satisfying the universal coefficient theorem (UCT) in terms of their K-theory and traces. I will discuss this result with a focus on the role of UCT. In the infinite setting, without the UCT, two unital Kirchberg algebras are isomorphic if and only they are KK-equivalent in a unit-preserving way. I'll discuss some results along these lines in the finite case.

THOMAS SINCLAIR, Purdue University
Computability of C-norms*

We will present joint work with Isaac Goldbring which combines techniques from continuous logic with the recent result MIP$^{\mathrm{co}}$ = coRE to answer a question of Fritz, Netzer, and Thom on the computability of the norm on $C^*(\mathbb{F}_2 \times \mathbb{F}_2)$.

CHARLES STARLING, Carleton University
Uniqueness theorems for combinatorial C-algebras*

Loosely speaking, a combinatorial C*-algebra is one defined in terms by generators and relations in some countable object, like a cancellative monoid, the paths in a directed graph, or the small category associated to a self-similar action. I will present uniqueness theorems akin to the classical Cuntz-Krieger uniqueness theorem, where injectivity of a *-homomorphism is equivalent to injectivity on a subalgebra generated by a subset of the generators.

AARON TIKUISIS, University of Ottawa
Basic homotopy lemmas via abstract classification

Basic homotopy lemmas, as a first approximation, are results saying that an approximately central unitary in a C*-algebra can be connected to the unit by a continuous path of approximately central unitaries. Going back to work of Bratteli, Elliott, Evans, and Kishimoto from the '90s, they feature heavily in the Gong-Lin-Niu classification of simple nuclear stably finite regular C*-algebras.

A different approach to classification (joint work with Carrión, Gabe, Schafhauser, and White) avoids the use of these basic homotopy lemmas, and produces an abstract classification of *-homomorphisms under minimal hypotheses. Moreover, the generality of our classification of *-homomorphisms allows us to prove a basic homotopy lemma in greater generality as an application. I will discuss our basic homotopy lemma and its relation to classification.

DAN URSU, York University
Non-conventional averaging in C-algebras*

Several important averaging properties have shown up in the theory of operator algebras, most notably the Dixmier averaging property and its variants, which deals with convex averages of elements in some unitary orbit. In joint work with Matthew Kennedy, expanding upon the work of Magajna in the theory of C*-convex averages, we develop a strong new averaging property and separation theorem, and use it to characterize when the intermediate subalgebra structure of a crossed product is entirely canonical. Progress-permitting, we will also give a sneak peek at some preliminary results using these same averaging techniques applied to the ideal structure of crossed products.