Harmonic Analysis & PDE
Analyse harmonique et EDP

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ALMAZ BUTAEV, University of the Fraser Valley
Minimizing some discrete energy functionals on a regular metric measure space

We will discuss a construction of discrete Dirichlet forms on Newtonian functions $N^{1,2}$ comparable to the upper gradient energy. Studying the Γ-limit of these Dirichlet forms we will talk about a boundary value problem associated with this limit. This is joint work with N. Shanmugalingam and L. Luo

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ANA ČOLOVIĆ, University of Missouri
Composition of Paraproducts

On the real line, we can decompose a product of two functions in $L^2(\mathbb{R})$ in the following way:

$$bf = \Pi_b(f) + \Pi_b^*(f) + \Pi_f(b),$$

where Π_b is the dyadic paraproduct. In 2013, Pott, Reguera, Sawyer and Wick classified the boundedness of compositons of paraproduct operators $\Pi_b\Pi_d^*$, $\Pi_b^*\Pi_d$, for different functions b and d, along with compositions of paraproducts with the dyadic martingales, in terms of joint conditions on the symbols b and d. We classify the remaining $\Pi_b\Pi_d$ operator.

JOSHUA FLYNN, MIT
Higher order boundary operators and trace inequalities on the Siegel domain and complex ball

We introduce conformally covariant boundary operators defined on the Heisenberg group and CR sphere, the boundaries of the Siegel domain and complex ball, respectively. We then establish for the Siegel domain and complex ball all associated higher order extension theorems of Caffarelli-Silvestre type and all higher order CR Sobolev trace inequalities.

ALPTEKIN GOKSAN, University of Toronto
A sharp condition for Békollé-Bonami weights to satisfy the reverse Hölder inequality

Békollé-Bonami B_p weights are the unit disc analogue of Muckenhoupt A_p weights and characterize L^p boundedness of the Bergman projection. It is well-known that A_p weights satisfy a reverse Hölder inequality, which brings with it a number of desirable properties such as self-improvement. On the other hand, B_p weights do not in general satisfy the reverse Hölder inequality, and this makes them harder to work with. Aleman, Pott and Reguera recently identified a condition under which B_p weights satisfy the reverse Hölder inequality. This condition requires that the weight be "almost constant" on the top halves of Carleson squares.

In this talk, we prove that being almost constant on top halves is a sharp condition for B_p weights to satisfy the reverse Hölder inequality. Moreover, we investigate the relationships between twelve conditions (including B_p and reverse Hölder) for weights on the unit disc. These conditions are known to be equivalent for weights on $\mathbb{R}^n$ (and are called A_∞ conditions in that case) and were recently studied by Duoandikoetxea, Martín-Reyes and Ombrosi in a more general context. We complete all missing implications and counterexamples between these twelve conditions, both for weights which are almost constant on top halves and for arbitrary weights on the unit disc.

JESSE HULSE, University of Manitoba
The Unified Transform Method: Beyond Circular or Convex Domains

A new transform-based technique that generalizes the Unified Transform Method is developed as a novel way to compute fluid flows and numerically solve boundary value problems for holomorphic functions and solutions to Laplacian. This work uses the Szegő kernel to extend previously developed transform pairs to bounded simply connected Lipschitz domains. Time permitting, two applications will be discussed.

NGUYEN LAM, Memorial University of Newfoundland
Sharp Stability of the Second-order Heisenberg Uncertainty Principle

In this talk, we present sharp stability estimates for the second-order Heisenberg Uncertainty Principle. We derive explicit lower and upper bounds for the associated sharp stability constants and determine their exact asymptotic limits as the dimension N tends to infinity. This is joint work with Anh Do, Guozhen Lu, and Lingxiao Zhang.

JUYOUNG LEE, Universitiy of British Columbia
Variational inequalities for two-parameter averages over tori

Variational inequalities have been extensively studied in various branches of mathematics. In particular, variational inequalities for averaging operators provide valuable insights into the pointwise convergence properties of averages. While maximal averaging operators offer similar results, variational inequalities yield more refined and powerful information regarding the behavior of averages. In this talk, we focus on a variational inequality for a two-parameter averaging operator. Indeed, a well-defined formulation of variational inequalities for two-parameter averages has not yet been established. We introduce a precise definition for two-parameter averages over tori and present a sharp boundedness result.

SULLIVAN MACDONALD, University of Toronto
Progress toward the Krzyz conjecture

The Krzyz conjecture is a long-standing open problem in complex analysis. Despite initially appearing simpler than related problems which have since been solved, such as the Bieberbach conjecture (now de Branges' Theorem), it remains open. If D is the unit disc and $\mathcal{B}^* = \{f \in \mathrm{Hol}(D) \mid 0 < |f| \leq 1 \text{ in } D\}$, it states that (1) $\sup_{f\in\mathcal{B}^*} |f^{(n)}(0)|/n! = 2/e$ for any $n \in \mathbb{N}$, and (2) the supremum is attained only by functions of the form $e^{i\theta} f_0(e^{i\eta} z)$, where $\theta, \eta \in \mathbb{R}$ and $f_0(z) = \exp((z^n - 1)/(z^n + 1))$.

In this talk we present recent work on the conjecture. Using techniques from classical harmonic analysis, we find new constraints on the singular inner functions which attain the supremum. It has long been known that extremal functions in $\mathcal{B}^*$ for the nth coefficient exist and are of the form

$$f(z) = \exp\left(\sum_{j=1}^{N} \lambda_j \frac{e^{i\theta_j} z - 1}{e^{i\theta_j} z + 1}\right)$$

for $N \leq n$, positive $\lambda_1, \ldots, \lambda_N$, and distinct $\theta_1, \ldots, \theta_N \in [0, 2\pi)$. Using oscillatory integral methods, we show that $N \geq cn$ for a universal constant $c > 0$. This marks modest progress toward proving the expected $N = n$. Various other new properties of extremal functions and their consequences will also be discussed.

Furthermore, we will report on progress related to other aspects of the conjecture.

ÁNGEL DAVID MARTÍNEZ MARTÍNEZ, CUNEF Universidad
On the monotonicity of the heat kernel

We will revisit a number of results about the monotonicity of the heat kernel on manifolds and their relation with well-known spectral inequalities. Based in joint work with Almut Burchard.

YUVESHEN MOOROOGEN, University of British Columbia
A large-scale variant of the Erdos similarity conjecture

Consider a sequence of real numbers increasing to infinity. How large can a subset of the real line be before it is forced to contain some affine image of that sequence? This question fits into a huge body of work in analysis and number theory concerned with constructing large sets that fail to contain prescribed structures. I will discuss recent progress on this question and comment on its connections with a now 50-year old open problem of Erdos.

CRISTIAN RIOS, University of Calgary
Characterizations of the Kakeya maximal conjecture in three dimensions

We present two characterizations of the Kakeya maximal problem in three-dimensional space in terms of a trilinear formulation. The bilinear equivalence due to Tao, Vargas, and Vega dates to 1998, and previous trilinear formulations due to Bourgain and Guth [BG] fall short of characterizing the problem. We introduce a weaker condition on the separation of the domains for the multilinear components than the transversality condition required in [BG] which proves to be equivalent to the whole linear conjecture. The proof also provides single-scale wavelet projection estimates which are sufficient to establish the conjecture.

SCOTT RODNEY, Cape Breton University

YURIJ SALMANIW, Cape Breton University
Well-posedness of aggregation-diffusion equations and systems with irregular kernels

In this talk, I will discuss some recent progress towards understanding properties of solutions to the aggregation-diffusion equation

$$\partial_t u = \Delta u + \nabla \cdot (u \nabla (W * u)),$$

where $W*\cdot$ denotes a spatial convolution with a kernel W on either $\mathbb{R}^d$ or $\mathbb{T}^d$. When W is sufficiently regular (e.g., $\nabla W \in L^\infty$), spatial derivatives can be transferred to W, and the existence and uniqueness of solutions follows from standard parabolic theory; in practice, kernels need not be differentiable, and one must use the regularity of u to deduce further information on the potential $W * u$. I will present some recent results regarding the existence and uniqueness of weak (and sometimes classical) solutions for merely bounded kernels (i.e., $W \in L^\infty$) under two regimes: a **small mass** regime, and an **arbitrary mass** regime with some additional structural requirements. I will then discuss how these results translate to the multi-species system with a matrix of bounded interaction kernels, highlighting what is known and where gaps remain. This is joint work with José Carrillo and Jakub Skrzeczkowski.

ERIC SAWYER, McMaster University
A comparison of trilinear testing conditions for the paraboloid Fourier extension and Kakeya conjectures in three dimensions

We compare the smooth Alpert testing condition for the paraboloid Fourier extension conjecture in RiSa3 to the modulated testing condition for the Kakeya conjecture in RiSa2. To this end, the modulated testing condition is converted to a certain restricted smooth Alpert testing condition for the paraboloid Fourier extension conjecture.

SHAHABODDIN SHAABANI, University of Toronto
A view from above on JN$_p((R)^n$

In this talk, we discuss the John–Nirenberg space JN$_p(\mathbb{R}^n)$ from the tent-space point of view. We show how the "tent perspective" on this space leads to a natural extension of the Riesz–Markov–Kakutani representation theorem to the upper half-space $\mathbb{R}^{n+1}_+$. We also demonstrate that this extension connects such representation theorems to well-known combinatorial geometric problems concerning intersection graphs of shapes and their chromatic numbers. If time permits, we will discuss an application related to the construction of functions in $\mathrm{JN}_p(\mathbb{R}^n)$.

IGNACIO URIARTE-TUERO, University of Toronto
Muckenhoupt A_p weights, BMO, distance functions and Hardy-Sobolev inequalities

Vasin (for $n = 1$) and Anderson, Lehrbäck, Mudarra, and Vähäkangas (for $n > 1$) provided a geometric characterization of the sets $E \subset \mathbb{R}^n$ so that $w = dist(\cdot, E)^{-\alpha}$ is a Muckenhoupt A_1 weight for some $\alpha > 0$. We provide a geometric characterization of the sets $E \subset \mathbb{R}^n$ (which we call median porous sets) so that $w = dist(\cdot, E)^{-\alpha}$ is a Muckenhoupt A_p weight for some $\alpha > 0$ (given any $1 < p \leq \infty$).

Given $1 < p \leq \infty$, we also find the precise range of exponents α so that $w = dist(\cdot, E)^{-\alpha} \in A_p$ (in analogy to the $p = 1$ case done by Anderson, Lehrbäck, Mudarra, and Vähäkangas).

With our characterization we prove that $\mathbb{R}^n \setminus E$ supports a Hardy-Sobolev inequality if E is an appropriate median porous set. All previous such results that we are aware of make the strictly stronger assumption that the set E is porous. The proofs rely on a new median-value characterization of BMO. Joint work with Marcus Pasquariello.

DIMITER VASSILEV, University of New Mexico
Regularity of solutions to non-local semilinear equations related to Sobolev type embeddings on homogeneous groups

I will present regularity and asymptotic decay results for positive solutions to certain non-local equations on homogeneous Lie groups. The equations include Schrödinger equations with suitable potential and the equations for the extremals of Sobolev type inequalities. In the latter case, the considered operator can be seen as generalization of nonlocal conformally invariant operators arising in conformal and sub-Riemannian geometry. However, we work in a setting where a Hörmander type condition might not hold.

KATJA VASSILEV, University of Chicago
One-dimensional wave kinetic theory

Kinetic theory aims to write effective equations for the statistical laws arising in microscopic systems with many degrees of freedom. Such equations, referred to as kinetic equations, have been studied dating back to the Boltzmann equation in the late 1800's and were later proposed for wave systems in the mid-1900's. Wave kinetic equations (WKEs) have been rigorously derived in dimension $d \geq 2$, but are less understood both physically and mathematically in dimension one. Here we consider the MMT (Majda, McLaughlin, and Tabak) model, a 1D dispersive model first proposed with the purpose of performing numerical studies on wave turbulence. This model encompasses various dispersive relations, some of which yield a trivial wave kinetic equation. For all dispersion relations we derive the (potentially trivial) WKE, and begin to answer the question of what the appropriate kinetic theory is in the setting when the WKE is trivial.

CHENJIAN WANG, The University of British Columbia
Pinned patterns and density theorems in $\mathbb{R}^d$

We consider the abundance property of pinned k-point patterns occurring in $E \subseteq \mathbb{R}^d$ with positive upper density $\delta(E)$. We show that for any fixed k-point pattern V, there is a set E with positive upper density such that E avoids all sufficiently large affine copies of V, with one vertex fixed at any point in E. However, we obtain a positive quantitative result, which states that for any fixed E with positive upper density, there exists a k-point pattern V, such that for any $x \in E$, a carefully chosen *pinned scaling factor set* has upper density $\geq \tilde{\varepsilon} > 0$, where constant $\tilde{\varepsilon}$ depends on k, d and $\delta(E)$.

JULIAN WEIGT, ICTP
Regularity of maximal functions in higher dimensions

The classical Hardy-Littlewood maximal function theorem states that the Hardy-Littlewood maximal operator is a bounded operator on $L^p(\mathbb{R}^d)$ if and only if $1 < p \leq \infty$. In 1997 Juha Kinnunen proved the corresponding result for the gradient of the maximal function, i.e. that the $L^p(\mathbb{R}^d)$-norm of the gradient of the maximal function is controlled by the $L^p(\mathbb{R}^d)$-norm of the gradient of the function if $1 < p \leq \infty$. However, he provides no counterexample in the endpoint $p = 1$, and so in 2004 Hajłasz and Onninen formally posed the question if the endpoint gradient bound also holds.

Many special cases, generalizations and variations of this problem have been explored, with partial success. The original question by Hajłasz and Onninen remains unanswered. We discuss recent progress in higher dimensions, based on the coarea formula, dyadic decompositions and the relative isoperimetric inequality. As a by-product we obtain a Vitali-type covering lemma for the boundary.

ALEXIA YAVICOLI, The University of British Columbia
The Erdős similarity problem for non-small Cantor sets

I will introduce the Erdős similarity problem, providing background and an overview of known partial results. I will then discuss a recent joint work with P. Shmerkin, in which we show that Cantor sets with positive logarithmic dimension satisfy the conjecture.