Combinatorial Algebraic Geometry
Géométrie algébrique combinatoire
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PATIENCE ABLETT, Queen Mary University London
Gotzmann's persistence theorem for smooth projective toric varieties

The Hilbert scheme is a scheme which parameterises subschemes living inside a fixed ambient space. By understanding the geometry of the Hilbert scheme itself, we can learn about the geometry of the subschemes we parameterise. In the case that our fixed ambient space is $\mathbb{P}^n$, this scheme is well understood using Gotzmann's regularity and persistence theorems. In this talk, we look at generalising Gotzmann's persistence result to the setting of any smooth projective toric variety.

MATT CARTIER, University of Pittsburgh
Computing the Invariant β for Certain Schubert Subvarieties

This talk concerns an invariant $\beta_Y(L)$ attached to a triple (X, Y, L), where X is an irreducible projective variety, $Y \subset X$ is a proper subscheme, and L is a big or ample line bundle on X. This invariant has arisen independently in several works, where it has been used to obtain new results in Diophantine geometry and mathematical physics. We will focus on the case where $X = \mathbb{G}(k, n)$, $L = O_X(1)$, and $Y \subset X$ a Schubert subvariety. We develop a method which, under suitable hypotheses, allows us to compute $\beta_Y(L)$ for any such Schubert subvariety Y. As a final result, using a different argument, we obtain a concise explicit formula for $\beta_Y(L)$ when Y is a special type of Schubert subvariety that we call a maximal rectangle Schubert subvariety (MR Schubert subvariety).

SANTIAGO ESTUPIÑÁN, University of Waterloo
A new shifted Littlewood-Richardson rule

As Littlewood-Richardson rules compute linear representation theory of symmetric groups, shifted Littlewood-Richardson rules compute analogous projective representation theory of symmetric groups. The first shifted Littlewood-Richardson rule is due to Stembridge (1989), building on a natural generalization by Sagan and Worley (1979/1984) of the jeu de taquin algorithm to shifted Young tableaux. We give a new shifted Littlewood-Richardson rule that requires consideration of fewer tableaux than Stembridge's rule and is provably faster on a family of structure coefficients. Our rule derives from applying old ideas of Lascoux and Schützenberger (1981) to the study of Haiman's mixed insertion (1989) and Serrano's shifted plactic monoid (2010).(Joint work with Oliver Pechenik.)

NATHAN GRIEVE, Carleton U./NTU/UQAM/U. Waterloo
Concepts of stability and positivity for big and nef line bundles, divisorial sheaves and divisors on the Zariski Riemann spaces

A key feature of the Neron-Severi spaces of divisor classes on the Zariski Riemann spaces is the absence of an ample cone. This highlights the question of defining K-stability invariants and measures of positivity for big and nef classes on projective varieties. The purpose of this talk is to report on recent progress in this direction. Some emphasis will be placed on my recent results about slope K-stability for big and nef divisors. Time permitting, I will report on my additional very recently obtained results which are in the general direction of the Riemann-Roch problem for birational divisors. For instance, this includes construction of Newton-Okounkov bodies for birational divisors and a concept of Kodaria-Iitaka dimension for fractional b-generalized log varieties.

KATRINA HONIGS, Simon Fraser University
McKay correspondence for reflection groups and derived categories

The classical McKay correspondence shows that there is a bijection between irreducible representations of finite subgroups G of $\mathrm{SL}(2,\mathbb{C})$ and the exceptional divisors of the minimal resolution of the singularity $\mathbb{C}^2/G$. This is a very elegant correspondence, but it's not at all obvious how to extend these ideas to other finite groups.

Kapranov and Vasserot, and then, later, Bridgeland, King and Reid showed this correspondence can be recast and extended as an equivalence of derived categories of coherent sheaves. When this framework is extended to finite subgroups of $\mathrm{GL}(2,\mathbb{C})$ generated by reflections, the equivalence of categories becomes a semiorthogonal decomposition whose components are, conjecturally, in bijection with irreducible representations of G. This correspondence has been verified in recent work of Potter and of Capellan for a particular embedding of the dihedral groups D_n in $\mathrm{GL}(2,\mathbb{C})$. I will discuss recent joint work verifying this decomposition in further cases.

NATHAN ILTEN, Simon Fraser University
Rational Curves in Projective Toric Varieties

I will discuss joint work with Jake Levinson in which we study embedded rational curves in projective toric varieties from a combinatorial perspective. We show that any degree d rational curve in a toric variety can be constructed from a special affine-linear map called a degree d Cayley structure. We characterize when the curves coming from a degree d Cayley structure are smooth and have degree d. We then use this to establish a bijection between the set of irreducible components of the Hilbert scheme whose general element is a smooth degree d curve, and so-called maximal smooth Cayley structures.

ELANA KALASHNIKOV, University of Waterloo
Tableaux Littlewood—Richardson rules for 2-step flags

The Abelian/non-Abelian correspondence gives rise to a natural basis for the cohomology of flag varieties, which - except for Grassmannians - is distinct from the Schubert basis. I will describe this basis and its multiplication rules, and explain how to relate it to the Schubert basis for two-step flag varieties. I will then explain how this leads to new tableaux Littlewood–Richardson rules for many products of Schubert classes. This is joint work (separately) with Wei Gu and Linda Chen.

JAKE LEVINSON, Université de Montréal
$\mathbb{A}^1$-degrees of twisted Wronski maps

In $\mathbb{A}^1$-enumerative geometry, the $\mathbb{A}^1$-degree of a finite map of varieties is given by counting points in a general fiber, weighted by, for each point, the class of its Jacobian determinant considered up to squares. This gives a sum in the Grothendieck-Witt ring of the base field, generalizing both the complex degree (absolute count of points in the fiber) and real topological degree (points weighted by the signs of their Jacobians) to arbitrary fields.

I will present some forthcoming work with Thomas Brazelton, in which we compute $\mathbb{A}^1$-degrees of Wronski maps with twisted real structures. Wronski maps arise in Schubert calculus and moduli of curves and have rich enumerative properties over both R and C; they measure ramification points of linear series on $\mathbb{P}^1$. By varying whether the ramification points are real or complex conjugate pairs, we vary the real structure of a Wronski-type family over $\overline{M_{0,n}}$. We describe how its $\mathbb{A}^1$-degree changes as the real structure is twisted.

CHRIS MANON, University of Kentucky
Polypytch varieties

Toric degenerations allow the combinatorial techniques of toric geometry to be applied to more general projective varieties. Escobar and Harada have defined a notion of wall-crossing in this context, where the moment polyhedra of different toric degenerations are connected by piecewise linear maps. Similar phenomena appear in the work of Rietsch and Williams, and Bossinger, Cheung, Magee, and Nájera Chávez on Newton-Okounkov bodies associated to compactifications of cluster varieties. In these settings, the piecewise linear maps reflect important aspects of the geometry and combinatorics of the associated variety. With Laura Escobar and Megumi Harada, we wrap the data of a collection of lattices related by piecewise-linear bijections together into a single semi-algebraic object, equipped with its own semialgebraic geometry, and notions of convexity and polyhedra. A generalized notion of a polyhedral fan in this setting then encodes compactifications of a certain affine variety. Various aspects of the geometry of this compactification can then be computed combinatorially. This is joint work with Escobar, Frias Medina, Harada, and Magee.

SHARON ROBINS, Carnegie Mellon University
Oda's Conjecture for Smooth Projective Toric Varieties of Lower Picard Rank

Oda's conjecture predicts the surjectivity of certain multiplication maps between linear systems on smooth projective toric varieties. Equivalently, it asserts a strong integer decomposition property for Minkowski sums of pairs of lattice polytopes. A key consequence of this surjectivity is that the smooth projective toric variety associated to the smooth polytope P is projectively normal with respect to all embeddings defined by the dilations of P. In this talk, I will survey known results on when the conjecture holds and present aspects of my work on this problem for toric varieties of lower Picard rank.

KAROLYN SO, Simon Fraser University
Gröbner Cones for Finite Type Cluster Algebras

Cluster algebras are a class of commutative algebras defined by a combinatorial iterative method. Consequently, many properties of cluster algebras may be studied through combinatorial tools. In the case of finite cluster type, the cluster algebra $\mathcal{A}$ is canonically a quotient of a polynomial ring by an ideal $I_{\mathcal{A}}$. By work of Ilten, Nájera-Chávez, and Treffinger, there exists a term order such that the initial ideal of $I_{\mathcal{A}}$ is the ideal generated by products of incompatible cluster variables. We study the Gröbner cone $\mathcal{C}_{\mathcal{A}}$ corresponding to this initial ideal. In joint work with Ilten, we construct distinguished elements of $\mathcal{C}_{\mathcal{A}}$ using compatibility degrees, and give explicit descriptions of the rays and lineality spaces of $\mathcal{C}_{\mathcal{A}}$ in terms of combinatorial models for cluster algebras of types A_n, B_n, C_n, D_n with a special choice of frozen variables, and in the case of no frozen variables. In this talk, I will discuss the main results in types A_n, B_n, and C_n.

SARA STEPHENS, Cornell University
Strictly Semistable Quasimaps to $\mathbb{P}^n$

One important moduli stack in enumerative geometry is that of ϵ-stable quasimaps, introduced by Ciocan-Fontanine, Kim, and Maulik for a broad class of GIT quotients. Varying the parameter ϵ yields a wall and chamber structure, giving rise to a family of intermediate Deligne-Mumford stacks that interpolate between the moduli of stable quasimaps and Kontsevich's stable maps. In this talk, I will describe an intrinsic perspective on the resulting wall-crossing phenomena via algebraic stacks, employing the framework of infinite-dimensional geometric invariant theory developed by Halpern-Leistner and collaborators. We construct an algebraic stack encoding an ϵ-stable quasimap wall crossing in the case where the semistable locus of the target is $\mathbb{P}^n$, and analyze necessary and sufficient filling conditions for this stack to admit a good moduli space.

TIANYI YU, LACIM
A positive combinatorial formula for the double Edelman–Greene coefficients

Lam, Lee, and Shimozono introduced the double Stanley symmetric functions to study the equivariant geometry of the affine Grassmannian. They showed that the double Edelman–Greene coefficients, the double Schur expansion coefficients of these functions, are Graham-positive. This positivity was later refined by Anderson. They further asked for an explicit combinatorial formula that manifests this positivity directly. We provide the first such formula, built from two combinatorial models: bumpless pipedreams and increasing chains in the Bruhat order. The key ingredients of our proof are a connection between these two models and a symmetry of increasing chains recently discovered by Sottile and Yu. This is a joint work with Jack Chen-An Chou.