Student Research Session

TANJIMA AKHTER & ADRIANA-STEFANIA CIUPEANU, University of Alberta and University of Manitoba
Preventing HPV-Induced Cervical Cancer in Alberta, Canada: A Mathematical Modelling study

Human Papillomavirus (HPV) is a widespread sexually transmitted disease responsible for nearly 99.7% cervical cancer. Despite extensive public health efforts, controlling HPV transmission remains challenging, and our objective was to explore factors contributing to the gap in achieving widespread prevention. This research used mathematical model to explore HPV infection and vaccination in Alberta, aiming to identify an effective program to reduce HPV-associated cervical cancer burden. We developed an age and sex stratified compartmental model based on the SIR framework, calibrated using Bayesian inference and Diffusive Nested Sampling in MATLAB. A key challenge was limited availability of male data due to minimal testing, lower awareness, and existing models under emphasizing men's role in HPV transmission, despite their significant contribution to the spread of the virus. By incorporating complex contact patterns and disease characteristics, the model assessed various vaccination scenarios and provided insights into long-term health outcomes, estimating reduction in cervical cancer incidence and quantifying the strategy's effectiveness. Our analysis revealed, implementing vaccination strategy with 90% coverage for individuals aged less than 20 and 40% coverage who aged 20+, across males and females, resulted in significant long-term benefits. Specifically, this approach led to 77% reduction in HPV prevalence among females aged 20+ and a 76% decrease in cervical cancer incidence over 52 years. These findings highlight the transformative potential of targeted vaccination strategies in reducing HPV cervical cancer, alleviating the disease burden, and saving lives. Such strategies can inform policy, raise awareness, and drive higher vaccination coverage for long-term public health benefits.

MICHAEL ASTWOOD, University of Manitoba
The Kepler Problem on Pseudo-Riemannian Surfaces

The generalized Kepler problem seeks to describe a Hamiltonian dynamical system determined by an arbitrary central potential. We introduce the classical Kepler problem as an instructive example and present original results on the generalized Kepler problem. We first demonstrate that the orbits of any Bertrand mechanical system on a pseudo-Riemannian surface of revolution are epitrochoids and provide explicit expressions for the orbital parameters. These results are complemented by numerical experiments using the recent symplectic integration methods of Tao and Pihajoki. We then construct analytic expressions for the super-integrals of the system, making explicit a result of Zagryadskii. Relevant concepts in differential geometry, geometric mechanics, and dynamical systems will be introduced.

KHALIL BESROUR, University of Ottawa
Introduction to Modular Forms and Modular Differential Equations

In this talk, tailored to a general audience, we introduce modular forms and we give some examples. We will also introduce modular differential equations of the form $y'' + F(z)y = 0$ on the upper half-plane , where F is a weight 4 modular form and we give explicit examples and solutions. Our method involves solving the associated Schwarzian equation $\{h, z\} = 2F(z)$, where $\{h, z\}$ denotes the Schwarzian derivative of a meromorphic function h. We will establish the conditions under which the solutions to this equation are modular functions for subgroups of the modular group, and we provide explicit expressions for these solutions in terms of classical modular functions. The primary tools in our analysis are the theory of equivariant functions on the upper half-plane, the theory of Riemann surfaces and some aspects of the representation theory of level 2 subgroups of the modular group.

LINH DINH, Dalhousie University
Contributions to the theory of Clifford-cyclotomic circuits

Circuit design is an important aspect of quantum computation theory, and the question of constructing a circuit that exactly represents a given arbitrary operator is prominent in the research area. The universal Clifford-cyclotomic gate sets are commonly used as the building blocks for circuit synthesis, consisting of the well-known Clifford gates and a rotation around the z-axis, where the angle is a rational multiple of π. This rotation, along with the Clifford gates, corresponds to certain subrings of the cyclotomic number fields generated by n-th roots of unity; and hence properties of algebra and number theory can be leveraged to solve this problem. In this talk, we will study the specific case of the 16th root of unity, also known as the Clifford+$\sqrt{T}$ gate set. Previous results on the Clifford+T gate set are well-understood within the field, and we have a concrete idea of the unitary group generated by Clifford+T, with and without ancillas. From here, we apply properties of determinants in block matrices to construct an embedding from the unitary group of Clifford+$\sqrt{T}$ into a specific subgroup of Clifford+T, where the exact synthesis algorithm can be performed without ancillas. This gives an improvement on the minimum number of ancillas required for Clifford-cyclotomic synthesis, which cuts down on the cost of circuit construction.

OWEN SHARPE, University of Waterloo
Prime Gradient Noise

Perlin noise is a technique for generating a smooth, random function from n-dimensional Euclidean space to the real numbers. The original algorithm described by Ken Perlin uses a small fixed table of vectors and a small predefined permutation table to generate randomness. These features induce undesirable periodicity in the generated function in large samples. We propose a method of generating a stream of pseudorandom vectors on the fly to replace these constructs, allowing us to generate an aperiodic function, all at relatively minimal cost. The key to the pseudorandom vector generation subalgorithm is a result of Vinogradov on prime numbers and equidistribution.

MATT SPRAGGE, Simon Fraser University
On the well-posedness of the Boltzmann equation for kinetic systems

In this talk we introduce the Boltzmann equation; a nonlinear partial integro-differential equation widely used to model the behaviour of dilute systems of interacting particles. Typical examples of such systems include dilute gases or plasmas, although Boltzmann-type equations have also been applied to study self-organizing behaviours within groups of living organisms, as well as the redistribution of wealth in simplified market economies. However, while its applications are, by now, widespread, the Boltzmann equation still presents numerous problems of great mathematical interest. In particular, the global existence and uniqueness of solutions has yet to be affirmed in general and it is this problem that we focus on in this presentation. A survey of known results as well as some recent progress regarding this problem will be discussed.

SCOTT WESLEY, Dalhousie University
Verifying and Simplifying Tietze Transformations

Invertible operations are described by groups. Many examples of groups, such as permutations and reversible circuits also admit nice combinatorial descriptions. These combinatorial descriptions often take the form of group presentations, which describe groups in terms of their generating elements, and the relations those elements satisfy. Unfortunately, identifying when two group presentations are isomorphic is known to be undecidable; that is, no algorithm can determine when arbitrary group presentations are isomorphic. In practice, people rely on Tietze transformations to build these isomorphisms by-hand. However, rigorously constructing proofs from Tietze transformations often proves to be tedious and time-consuming.

In this talk, we propose a new framework for building isomorphisms from Tietze transformations. We start by viewing Tietze transformations as a proof system for establishing group isomorphisms. From this perspective, the validity of a Tietze-style proof is decidable, whereas the existence of a Tietze-style proof is undecidable. On top of this foundation, we show that basic semantic arguments and compositional reasoning can be added to the proof system, without losing the decidability of proof validity. We then implement a software package, which we call Tietze, to automatically validate proofs in this proof system. We demonstrate the effectiveness of Tietze to solve real problems, and discuss directions for future work, such as proof visualization and interactive theorem proving.