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A second-order accurate numerical scheme for the Poisson-Nernst-Planck-Navier-Stokes (PNPNS) system

In this talk, I will present a second order accurate (in both time and space) numerical scheme for the Poisson-Nernst-Planck-Navier-Stokes system, which describes the ion electro-diffusion in fluids. In particular, the Poisson-Nernst-Planck equation is reformulated as a non-constant mobility gradient flow in the Energetic Variational Approach. The marker and cell finite difference method is chosen as the spatial discretization, which facilitates the analysis for the fluid part. In the temporal discretization, the mobility function is computed by a second order extrapolation formula for the sake of unique solvability analysis, while a modified Crank-Nicolson approximation is applied to the singular logarithmic nonlinear term. Nonlinear artificial regularization terms are added in the chemical potential part, so that the positivity-preserving property could be theoretically proved. Meanwhile, a second order accurate, semi-implicit approximation is applied to the convective term in the PNP evolutionary equation, and the fluid momentum equation is similarly computed. In addition, an optimal rate convergence analysis is provided, based on the higher order asymptotic expansion for the numerical solution, the rough and refined error estimate techniques. The following combined theoretical properties have been established for the second order accurate numerical method: (i) second order accuracy, (ii) unique solvability and positivity, (iii) total energy stability, and (iv) optimal rate convergence. A few numerical results are displayed to validate the theoretical analysis.