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Runge-Type Approximation Theorem for Banach-valued H^∞ Functions on a Polydisk

Let $\mathbb{D}^n \subset \mathbb{C}^n$ be the open unit polydisk, $K \subset \mathbb{D}^n$ be an n-ary Cartesian product of planar sets, and $\widehat{U} \subset \mathfrak{M}^n$ be an open neighbourhood of the closure $\bar{K}$ of K in $\mathfrak{M}^n$, where $\mathfrak{M}$ is the maximal ideal space of the algebra H^∞ of bounded holomorphic functions on $\mathbb{D}$. Let X be a complex Banach space and $H^\infty(V, X)$ be the space of bounded X-valued holomorphic functions on an open set $V \subset \mathbb{D}^n$. We show that any $f \in H^\infty(U, X)$, where $U = \widehat{U} \cap \mathbb{D}^n$, can be uniformly approximated on K by ratios h/b, where $h \in H^\infty(\mathbb{D}^n, X)$ and b is the product of interpolating Blaschke products such that $\inf_K |b| > 0$. Moreover, if $\bar{K}$ is contained in a compact holomorphically convex subset of $\widehat{U}$, then h/b above can be replaced by h for any f. The results follow from a new constructive Runge-type approximation theorem for Banach-valued holomorphic functions on open subsets of $\mathbb{D}$ and extend the fundamental results of Suarez on Runge-type approximation for analytic germs on compact subsets of $\mathfrak{M}$. They can also be applied to the long-standing corona problem which asks whether $\mathbb{D}^n$ is dense in the maximal ideal space of $H^\infty(\mathbb{D}^n)$ for all $n \geq 2$.