Incidence Problems in Analysis

(Org: **Ryan Bushling** (University of Washington), **William O'Regan** (University of British Columbia) and/et **Bobby Wilson** (University of Washington))

PAIGE BRIGHT, Univeristy of British Columbia

Dual Furstenberg Sets

Recently, Ren and Wang resolved the Furstenberg set problem in the plane; a fractal version of the Kakeya problem. In the plane, via a tool known as point-line duality, the Furstenberg set problem is directly related to a problem often referred to as the dual Furstenberg set problem. The focus of this talk will be the dual Furstenberg set problem in higher dimensions, motivated by problems in/applications to projection theory. This is joint work with Yuqiu Fu and Kevin Ren.

ALEX COHEN, MIT

Branching structure in phase space

Let (p_j, ℓ_j) be a collection of point-line pairs with ℓ_j passing through p_j. We associate to this configuration a *branching function* $f(x, y, z)$ of three variables which measures how much the configuration concentrates in rectangles of various side ratios. Geometrical information about incidences can be phrased as algebraic information about f. This framework provides a new way to ask and answer questions about two dimensional continuous incidence geometry.

Joint with Cosmin Pohoata and Dimitrii Zakharov.

JACOB B. FIEDLER, University of Wisconsin-Madison

Universal sets for pinned distances

An important problem in geometric measure theory is bounding the size of pinned distance sets $\Delta_x Y = \{|x-y| : y \in Y\}$. We discuss recent work on this problem in the plane which shows that, as long as the pin x satisfies certain properties, the pinned distance set of Y at x will be as large as possible. In particular we show that any compact AD-regular set X of dimension more than 1 has the property we call universality: for any Borel Y, there is an x in X such that the pinned distance set of Y at x has maximum Hausdorff dimension, i.e. $\min\{1, \dim_H(Y)\}$. We will also discuss improved bounds when no regularity assumption is made on the pin set. This is based on joint work with Don Stull.

ROBERT FRASER, Wichita State University

A Framework for constructing large sets without configurations

We describe a framework introduced in 2020 for constructing subsets of $\mathbb{R}^n$ of large Hausdorff dimension that avoid certain kinds of configurations. We present some possible future directions for this framework.

TERENCE HARRIS, UW Madison

Exceptional sets for length under restricted families of projections onto lines in $\mathbb{R}^3$

I will present the following theorem: If $A \subseteq \mathbb{R}^3$ is a Borel set of Hausdorff dimension $\dim A > 1$, then the orthogonal projection of A onto the line spanned by $(\cos\theta, \sin\theta, 1)$ has positive length for all θ outside a set of Hausdorff dimension at most $\frac{3-\dim A}{2}$.

IZABELLA ŁABA, UBC

Incidence questions in p-adic geometry

Let $R = \mathbb{Z}/p^k\mathbb{Z}$, where p is a prime. For $k = 1$, R is a finite field, and there is a significant body of work on incidence geometry in R^n in this case. For $k \geq 2$, R is only a ring and not a field. Incidence questions in this case have new features: for example, multiple scales are present, and two non-parallel lines may intersect in more than one point depending on their angle. Major recent advances include the results of Dhar, Dvir, and Arsovski on the Kakeya problem over rings $\mathbb{Z}/N\mathbb{Z}$. I will discuss some new work on incidence questions in this setting. (Based on joint work with Charlotte Trainor and with Hailong Dao, Manik Dhar, and Ben Lund.)

CALEB MARSHALL, University of British Columbia
Pinned Dot Product Set Estimates

Choosing a fractal subset $A \subset \mathbb{R}^n$ and points $a, x \in \mathbb{R}^n$, the Falconer pinned dot product set problem asks how large the associated pinned dot product sets

$$\Pi_x^a(A) := \{\alpha \in \mathbb{R} : (a - x) \cdot y = \alpha, \text{ for some } y \in A\},$$

must be, relative to the Hausdorff dimension of A and choice of points $a, x \in \mathbb{R}^n$.

We discuss our new method for studying this problem. In particular, we determine lower bounds on the Hausdorff dimension of A which guarantee that $\Pi_x^a(A)$ is large in some quantitative sense for many $a \in A$. Our proof method is robust enough to show that the set $\Pi_x^a(A)$ has large Hausdorff dimension, positive measure, or nonempty interior, so long as we assume that the dimension of A is large enough. The proof utilizes both classical and recent results on orthogonal and radial projection theory. We also discuss possible extensions and open questions raised by this method.

This talk is based on upcoming joint work with S. Senger (Missouri State) and P. Bright (UBC).

ALEX MCDONALD, Kennesaw State University
Prescribed projections and efficient coverings of sets by curves

A remarkable result of Davies shows that an arbitrary measurable set in the plane can be covered by lines "efficiently", in the sense that the parts of the lines not needed form a set of measure zero in the plane. This theorem has an equivalent dual formulation which says that one can find a single set in the plane with given "prescribed" projections in almost every direction, up to measure zero errors. We extend these results to a non-linear setting and prove that a set in the plane can be covered efficiently by translates of a single curve satisfying a mild curvature assumption.

YUVESHEN MOOROOGEN, University of British Columbia
A large-scale variant of the Erdos similarity conjecture

Consider a sequence of real numbers increasing to infinity. How large can a subset of the real line be before it is forced to contain some affine image of that sequence? This question fits into a huge body of work in analysis and number theory concerned with constructing large sets that fail to contain prescribed structures. I will discuss recent progress on this question and comment on its connections with a now 50-year old open problem of Erdos.

RAANI K. S. SENTHIL, IISER Berhampur
Distribution of distances in quasi-regular sets

In 1990, Strichartz introduced the notion of quasi regular sets. An $s-$dimensional set $E \subset [0,1]^d$ is said to be quasi-regular if there exists $\kappa > 0$ such that

$$\liminf_{r \to 0} \frac{1}{(2r)^s} \mathbb{H}^s(E \cap B(x;r)) \geq \kappa$$

for $\mathbb{H}^s-$almost every $x \in E$. Strichartz further studied the Fourier asymptotics of measures supported on these sets. In this talk we discuss the role of their Fourier asymptotics in determining the nature of the distances in quasi-regular sets. This is based on joint work with Prof. Malabika Pramanik.

DONALD M. STULL, University of Chicago
Exceptional sets for orthogonal directions

It is well known that if $A \subseteq \mathbb{R}^n$ is an analytic set of Hausdorff dimension a, then $\dim_H(\pi_V A) = \min\{a, k\}$ for a.e. $V \in G(n,k)$, where π_V is the orthogonal projection of A onto V. In this talk we discuss how large the exceptional set

$$\{V \in G(n,k) \mid \dim_H(\pi_V A) < s\}$$

can be for a given $s \leq \min\{a, k\}$. We improve previously known lower bounds on the dimension of the exceptional set, and we show that our estimates are sharp for $k = 1$ and for $k = n - 1$. This is joint work with Peter Cholak, Marianna Csornyei, Neil Lutz, Patrick Lutz and Elvira Mayordomo.