Harmonic Analysis and Geometric Measure Theory
(Org: **Sean Douglas**, **Caleb Marshall** and/et **Yuveshen Mooroogen** (University of British Columbia))

BODAN ARSOVSKI, IAS

TAINARA BORGES, Brown University
Bounds for bilinear averages and associated maximal functions

Let S^{2d-1} be the unit sphere in $\mathbb{R}^{2d}$, and σ_{2d-1} the normalized spherical measure in S^{2d-1}. The (scale t) bilinear spherical average is given by

$$\mathcal{A}_t(f,g)(x) := \int_{S^{2d-1}} f(x-ty)g(x-tz)\,d\sigma_{2d-1}(y,z).$$

There are geometric motivations to study bounds for such bilinear spherical averages, in connection to the study of some Falconer distance problem variants. Sobolev smoothing bounds for the operator

$$\mathcal{M}_{[1,2]}(f,g)(x) = \sup_{t\in[1,2]} |\mathcal{A}_t(f,g)(x)|$$

are also relevant to get bounds for the bilinear spherical maximal function

$$\mathcal{M}(f,g)(x) := \sup_{t>0} |\mathcal{A}_t(f,g)(x)|.$$

In a joint work with B. Foster and Y. Ou, we put that in a general framework where S^{2d-1} can be replaced by more general smooth surfaces in $\mathbb{R}^{2d}$, and one can allow more general dilation sets in the maximal functions: instead of supremum over $t>0$, the supremum can be taken over $t\in\tilde{E}$ where $\tilde{E}$ is the set of all scales obtained by dyadic dilation of fixed set of scales $E\subseteq[1,2]$.

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PAIGE BRIGHT, University of British Columbia
A Continuum Erdős–Beck Theorem

In discrete geometry, a classical result of Beck roughly shows that given a set $X \subset \mathbb{R}^n$ that is not too concentrated on any line, there are many (i.e. roughly $\gtrsim |X|^2$) distinct lines that contain at least 2 points of X. In 2022, Orponen, Shmerkin, and Wang proved a continuum version of Beck's theorem using tools from geometric measure theory. In this talk, we will present a continuum variant of Beck's theorem, known as the Erdős–Beck theorem, obtained in joint work with Caleb Marshall.

RYAN BUSHLING, University of Washington
An Integral Identity with Applications to Convex Sets

We propose a notion of "orientation" for $(n-1)$-rectifiable sets in $\mathbb{R}^n$. Using the classical methods of geometric measure theory, we prove that the integral of a certain Riesz-type kernel over these "oriented" sets is an absolute constant, from which a formula for surface measure immediately follows. Geometric interpretations are given, and the solution to a geometric variational problem characterizing the family of convex sets follows as a corollary.

EMILY CASEY, University of Washington
Anisotropic singular integrals and rectifiability

Since the work of Mattila and Preiss in 1995, it's been known that for a Radon measure with reasonable density assumptions, the almost everywhere existence of principal values of the Riesz transform is equivalent to the measure being rectifiable. In ongoing work with M. Goering, T. Toro, and B. Wilson, we prove a version of this result of Mattila and Preiss in a rough Riemannian setting. In this talk, we discuss the geometric techniques used in proving the almost everywhere existence of principal values for smooth Calderon Zygmund kernels for rectifiable measures, even when the kernel is not of convolution type.

ANGEL CRUZ, University of British Columbia
Fourier Dimension and Trasnlation-Invariant Linear Equations

Sets of large Fourier dimension often contain certain configurations that their counterparts of the same Hausdorff dimension may miss. In this talk we present an application of the strategies introduced by Yiyu Liang and Malabika Pramanik to confirm the intuition that any set of large Fourier dimension contains certain nontrivial patterns, in the form of solutions of translation-invariant linear equations with integer coefficients.

JACOB DENSON, University of Wisconsin, Madison
A Characterization of Boundedness For Multipliers of Spherical Harmonic Expansions

Any function on the sphere S^d has a unique orthogonal expansion of the form $f = \sum_{k=0}^{\infty} H_k f$, where $H_k f$ is a spherical harmonic of degree k. For a function $m : (0, \infty) \to \mathbb{C}$, define a family of operators T_R on S^d for each $R > 0$ by setting $T_R f = \sum m(k/R) H_k f$. Such operators are called 'multiplier operators for spherical harmonic expansions', or 'zonal convolution operators'. For $d \geq 4$, and a limited range of L^p spaces, we find necessary and sufficient conditions for the operators $\{T_R\}$ to be uniformly bounded on $L^p(S^d)$. As a consequence, we prove a 'transference principle', that the operators $\{T_R\}$ are uniformly bounded on $L^p(S^d)$ if and only if the radial Fourier multiplier operator on $\mathbb{R}^d$ with symbol $m(|\cdot|) : \mathbb{R}^d \to \mathbb{C}$ is bounded on $L^p(\mathbb{R}^d)$. Such necessary and sufficient conditions, and the resulting transference principle, are unknown for analogous spectral multipliers on any compact manifold and any $p \neq 2$. Our proof methods involve using the theory of Fourier integral operators to find variable-coefficient analogues of strategies for bounding radial Fourier multiplier operators on Euclidean space.

MEET FOR GROUP DINNER,

SEAN DOUGLAS, University of British Columbia
Chain Rule For Weighted Triebel-Lizorkin Spaces

In this talk, we establish a fractional chain rule in the context of weighted Triebel-Lizorkin spaces under various smoothness conditions. This result notably extends the fractional chain rule to weighted Sobolev spaces with an integrability index less than one. Additionally, we determine an explicit relationship between the smoothness index, the integrability index, and the choice of weights. Furthermore, the fractional chain rule for smoothness index $0 < s < 1$ is extended to a normed fractional Faà di Bruno inequality for $s > 0$ within the framework of Sobolev spaces.

IZABELLA ŁABA, UBC
A short survey of integer tilings

A set $A \subset \mathbb{Z}$ tiles the integers by translations if there is a set $T \subset \mathbb{Z}$ such that every integer $n \in \mathbb{Z}$ has a unique representation $n = a + t$ with $a \in A$ and $t \in T$. The main open question regarding integer tilings is the Coven-Meyerowitz conjecture, providing a tentative characterization of finite tiles. We will survey some of the recent developments and open questions in this area, including a recent joint result with Itay Londner where we prove the Coven-Meyerowitz tiling conditions for a new class of tilings.

AKOS MAGYAR, University of Georgia

K.S. SENTHIL RAANI, IISER Berhampur
Sets containing all sufficiently large distances

In 1986, Falconer-Marstrand, Furstenberg-Katznelson-Weiss and Bourgain improved Boardman results independently that the unbounded d-dimensional sets $A \subset \mathbb{R}^d$ with positive asymptotic density admits sufficiently all large distances. In this talk we introduce the notion of well-distributed sets with s-density; an s-dimensional set is well distributed if its high density scales are not too sparsely located on $\mathbb{R}$. Suppose s is close to d. We prove that a well distributed set $A \subset \mathbb{R}^d$ with s-density, admits sufficiently all large distances. This is based on the joint work with Prof. Malabika Pramanik.

SHAHABODDIN SHAABANI, Concordia University
The Operator Norm of Paraproducts on Bi-parameter Hardy Spaces

In this talk, we discuss the recent work on the operator norm of bi-parameter paraproducts on bi-parameter Hardy spaces. A paraproduct is a bi-linear form arising in the product of two functions both expanded in either a wavelet basis such as the Haar or in the Littlewood-Paley pieces. In the one-parameter theory, the frequency interactions in the product of two functions are divided into either low-low, low-high or high-high interactions, and each gives rise to a bi-linear form called a one parameter paraproduct. These forms behave much better than the product itself and for them the Holder's inequality extends to the full range of Hardy spaces.

In my recent work, it is shown that for $0 < p, q, r < \infty$, with $\frac{1}{q} = \frac{1}{p} + \frac{1}{r}$, the operator norm of the dyadic bi-parameter paraproduct of the form

$$\pi_g(f) := \sum_R \langle g, h_R \rangle \langle f \rangle_R h_R,$$

from the bi-parameter dyadic Hardy space $H^p_d(\mathbb{R} \otimes \mathbb{R})$ to $H^q_d(\mathbb{R} \otimes \mathbb{R})$ is comparable to $\|g\|_{H^r_d(\mathbb{R}\otimes\mathbb{R})}$. In the above, the some is taken over all dyadic rectangles in the plane, h_R is the bi-parameter Haar wavelet supported on R, and $\langle f \rangle_R$ is the average of f over R. It is also proved that for all $0 < p < \infty$, there holds

$$\|g\|_{BMO_d(\mathbb{R}\otimes\mathbb{R})} \simeq \|\pi_g\|_{H^p_d(\mathbb{R}\otimes\mathbb{R}) \to H^p_d(\mathbb{R}\otimes\mathbb{R})},$$

where $BMO_d(\mathbb{R} \otimes \mathbb{R})$, stands for the dyadic product BMO space. Similar results are obtained for bi-parameter Fourier paraproducts of the same form.

PABLO SHMERKIN, UBC

Restricted projections and self-similar sets

It is generally expected that "structured fractals" such as self-similar sets have regular geometric behaviour. In particular, one expects that projecting a self-similar set preserves dimension, unless there is some obvious obstruction. I will present joint work with Amir Algom in which we extend results of Hochman-Shmerkin and Falconer-Jin in this direction. Recent results on restricted projections play a key role.

KRYSTAL TAYLOR, Ohio State University

Projections and Favard length in a nonlinear setting

Projections detect information about the size, geometric arrangement, and dimension of sets. In recent years, there has been significant interest in determining the rates of decay of the classical Favard length (or average orthogonal projection length) for various fractal sets. For orthogonal projections, quantitative estimates rely on a separation condition: most points are well-differentiated by most projections. It turns out that this idea also applies to a broad class of nonlinear projection-type operators satisfying a transversality condition. This begs the question of obtaining quantitative upper and lower bounds for decay rates for nonlinear variants of Favard length, including Favard curve length (as well as a new generalization to higher dimensions, called Favard surface length) and visibility measurements associated with radial projections. As one application, we consider the decay rate of the Favard curve length of generations of the four corner Cantor set. Our upper bound utilizes the seminal work of Nazarov, Peres, and Volberg, while energy techniques play a role in achieving a lower bound. This talk is based on joint works with Cladek and Davey and with Bongers.

RODOLFO TORRES, University of California, Riverside

EXTRAPOLATION OF COMPACTNESS FOR CERTAIN PSEUDODIFFERENTIAL OPERATORS

The extrapolation result of Rubio de Francia has become a powerful tool to extend the weighted boundedness of an operator from a particular weighted Lebesgue space into others. This classical theorem has been extended to many contexts over the years and found many useful application and, more recently, versions to extrapolate compactness have been studied by several authors too. We will provide a simple alternative version of such extrapolation of compactness results and present a novel application to a class of pseudodifferential operators, establishing their compactness on weighted Lebesgue spaces. This is joint work with María Jesús Carro and Javier Soria.

IGANCIO URIARTE-TURO, University of Toronto

Two weight norm inequalities for singular and fractional integral operators in $\mathbb{R}^n$

I will give an overview of the history and some applications, and report on some of the recent progress on the two weight problem for singular integral operators in $\mathbb{R}^n$. In particular, I will elaborate on the original Nazarov-Treil-Volberg conjecture, its proof for the Hilbert transform, partial results for other Calderón-Zygmund operators in higher dimensions, necessity (or lack thereof) of various conditions, extensions to Tb theorems, stability results of bump conditions vs testing conditions, etc.). The talk refers to joint works with Alexis, Grigoriadis, Lacey, Luna-Garcia, Paparizos, Sawyer, and Shen.

ALEXIA YAVICOLI, The University of British Columbia

The Erdős similarity problem for non-small Cantor sets

The Erdős similarity conjecture posits that any infinite set of real numbers cannot be affinely embedded into every measurable set of positive Lebesgue measure. I will show that Cantor sets of "positive logarithmic dimension" satify the Erdős similarity conjecture. This talk is based on a work in progress with Pablo Shmerkin.

JOSH ZAHL, UBC
curve tangencies and maximal functions

I will discuss a class of maximal operators that arise from averaging functions over thin neighborhoods of curves in the plane. Examples of such operators are the Kakeya maximal function and the Wolff and Bourgain circular maximal functions. To understand the behavior of these operators, we need to study the possible intersection patterns for collections of curves in the plane: how often can these curves intersect, how often can they be tangent, and how often can they be tangent to higher order?

JUNQIANG ZHANG, China University of Mining and Technology-Beijing
On Odd Normal Numbers

A real number x is considered normal in an integer base $b \geq 2$ if its digit expansion in this base is "equitable", ensuring that for each $k \geq 1$, every ordered sequence of k digits from $\{0, 1, \ldots, b-1\}$ occurs in the digit expansion of x with the same limiting frequency. Borel's classical result asserts that Lebesgue-almost every $x \in [0, 1]$ is normal in every base $b \geq 2$. We consider the set $N(O, E)$ of reals that are normal in odd bases but not in even ones. It is known that this set has full Hausdorff dimension but zero Fourier dimension. The latter condition means that $N(O, E)$ cannot support a probability measure whose Fourier transform has power decay at infinity. Our main result is that $N(O, E)$ supports a Rajchman measure μ, whose Fourier transform $\widehat{\mu}(\xi)$ approaches 0 as $|\xi| \to \infty$ by definiton. Moreover, the decay rate of $\widehat{\mu}$ is essentially optimal, subject to the constraints of its support. The methods draw inspiration from the number-theoretic results of Schmidt and a construction of Lyons. As a consequence, $N(O, E)$ emerges as a set of multiplicity, in the sense of Fourier analysis. This addresses a question posed by Kahane and Salem in the special case of $N(O, E)$. This is a joint work with Professor Malabika Pramanik.

JUNJIE ZHU, University of British Columbia
Hausdorff dimension and quadratic Roth

Many results in harmonic analysis and geometric measure theory ensure the existence of geometric configurations under the largeness of sets, which are sometimes specified using the ball condition and Fourier decay. Recently, Kuca, Orponen, Sahlsten, and Bruce, Pramanik proved a Sarkozy-like theorem, which removes the Fourier decay condition and shows that sets with large Hausdorff dimensions contain two-point patterns. The existence of a three-point configuration relying solely on the Hausdorff dimension remains intractable so far. I am reporting my ongoing work in this direction.