
KATHIE CAMERON, Wilfrid Laurier University

Frozen Colourings

A k -colouring of a graph is an assignment of at most k colours to its vertices so that the ends of each edge get different colours. We consider the question: When it is possible to obtain any k -colouring from any other by changing the colour of one vertex at a time, while always having a k -colouring? This is equivalent to asking whether the "reconfiguration graph" is connected: The reconfiguration graph of the k -colourings, denoted $R_k(G)$, is the graph whose vertices are the k -colourings of G , and two colourings are adjacent in $R_k(G)$ if they differ in colour on exactly one vertex.

A k -colouring is called frozen if there is no vertex whose colour can be changed so that the result is still a k -colouring. A frozen colouring corresponds to an isolated vertex of the reconfiguration graph. Equivalently, a frozen k -colouring is a partition of the vertices into at most k sets, each of which is both independent and dominating. The terms fall colouring and indominable have also been used. We have found several new classes of graphs with frozen colourings and an operation which transforms a k -chromatic graph with a frozen $(k + 1)$ -colouring into a $(k + 1)$ -chromatic graph with a frozen $(k + 2)$ -colouring, without using the "join" operation or adding universal vertices. I will discuss the recently resolved problem: For which values of k and t does there exist a k -colourable graph with no induced path on t vertices with a frozen $(k + 1)$ -colouring?

This is joint work with Manoj Belavadi and Elias Hildred.