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Vertex-critical graphs in co-gem-free graphs

In this talk, we will show that there are only finitely many k-vertex-critical (co-gem, H)-free graphs for all k when H is any graph of order 4 by showing finiteness in the three remaining open cases, those are the cases when H is $2P_2$, $K_3 + P_1$, and K_4. Here a graph G is k-vertex-critical if $\chi(G) = k$ but $\chi(G - v) < k$ for all $v \in V(G)$ and (G, H)-free if it contains no induced subgraph isomorphic to G or H. The co-gem is the the disjoint union of a path of order 4 and a single vertex. For the first two cases we actually prove the stronger results:

- There are only finitely many k-vertex-critical (co-gem, paw$+P_1$)-free graphs for all k and that only finitely many k-vertex-critical (co-gem, paw$+P_1$)-free graphs for all $k \geq 1$.
- There are only finitely many k-vertex-critical (co-gem, P_5, $P_3 + cP_2$)-free graphs for all $k \geq 1$ and $c \geq 0$.

Our results imply the existence of simple polynomial-time certifying algorithms to decide the k-colourability of (co-gem, H)-free graphs for all k and all H of order 4 by searching for the vertex-critical graphs as induced subgraphs. As time allows, we will sketch some of the new ideas used in our proofs, including an application of Sperner's Theorem on the number of antichains in a partially ordered set.