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Quaternary Legendre pairs of even length

One of the most famous open problems in discrete mathematics is Paley's 1933 conjecture that there is a Hadamard matrix of order $n > 2$ if and only if n is a multiple of 4. It has long been known that this conjecture would follow from the existence of a pair of binary Legendre sequences for every odd length. It has recently been shown that this conjecture would also follow from the existence of a pair of quaternary Legendre sequences for every even length.

We use finite fields to give the first general constructions of quaternary Legendre sequences of even length. In particular, we modify a classical construction due to Szekeres to show that there is a quaternary Legendre sequence of even length $(q - 1)/2$ for every prime power q congruent to 1 modulo 4.

This is joint work with Thomas Pender.