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A Closest Point Method for PDEs on Manifolds with Interior Boundary Conditions for Geometry Processing

Solving partial differential equations (PDEs) on manifolds is fundamental to many geometry processing tasks, such as diffusion curves on surfaces, geodesic computations, tangent vector field design, and reaction-diffusion textures. These PDEs often involve boundary conditions prescribed at points or curves on the manifold's interior or along the geometric boundary of an open manifold.

We present a robust extension of the closest point method (CPM) for handling interior boundary conditions. The CPM reformulates the manifold PDE as a volumetric PDE in the Cartesian embedding space, requiring only the closest point representation of the manifold. This approach inherently supports open or closed manifolds, orientable or not, and of any codimension. To address interior boundary conditions, we derive a technique that implicitly partitions the embedding space across interior boundaries, modifying finite difference and interpolation stencils to respect these partitions while preserving second-order accuracy.

Our method includes an efficient sparse-grid implementation and scalable numerical solver capable of handling tens of millions of degrees of freedom, enabling solutions on complex manifolds. We demonstrate the convergence and accuracy of our approach using model PDEs and showcase applications to a range of geometry processing problems.

This is joint work with Nathan King (University of Waterloo), Haozhe Su (Lightspeed Studios), Mridul Aanjaneya (Rutgers University), and Christopher Batty (University of Waterloo).