JAXON SHUMAKER, University of Oregon
Classifying monogenic quartic orders

An order is a subring of the ring of integers of a number field. We say an order is *monogenic* if it is generated by a single element as an algebra over $\mathbb{Z}$. Following Bèrczes, Evertse, Győry, we consider two types of monogenic orders. A monogenic order is *type I* if there are two monogenizers ξ and β and there exist integers $a, b, c, d \in \mathbb{Z}$, with $c \neq 0$, such that $\beta = \dfrac{a\xi + b}{c\xi + d}$ and $ad - bc = \pm 1$. Additionally, a monogenic order is said to be type II if, for two generators ξ and β, there exist $f, g \in \mathbb{Z}[T]$, such that, $\deg(f) = \deg(g) = 2$ and $f(\xi) = \beta$, $g(\beta) = \xi$. It is proven by Bérczes, Evertse, and Győry that for a given number field, with certain conditions on the Galois group of the number field, nearly all the monogenic orders having at least two distinct generators, are type I or type II.

The work in this presentation is motivated by the following question "Do there exist monogenic quartic orders that are neither type I nor type II, and if so, can an explicit bound on the number of exceptional orders in a given quartic number field be given?" We use some of Akhtari's explicit results about index form equations in quartic number fields, to explore this question.