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Boundedness of average rank of elliptic curves ordered by the coefficients

In arithmetic statistics, elliptic curves are typically ordered by the naive height, defined for $E_{A,B} : y^2 = x^3 + Ax + B$ as $H(E_{A,B}) = \max\{4|A|^3, 27B^2\}$, which effectively orders curves by the size of their roots. In this paper, we consider an alternative height function, $h(E_{A,B}) = \max\{|A|, |B|\}$, ordering elliptic curves by the magnitudes of their coefficients. We demonstrate that, under this new height function, the average size of the 2-Selmer group is bounded above by 3, aligning with the findings of Bhargava and Shankar under the naive height. We study the 2-Selmer group by analyzing integral binary quartic forms within non-uniformly expanding regions defined by the height function. Developing a new technique, we count and establish the equidistribution of lattice points in these spaces, overcoming challenges where standard methods fall short.