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Complexity of trust-region methods in the presence of unbounded Hessian approximations

We extend traditional complexity analyses of trust-region methods for unconstrained, possibly nonconvex, optimization. Whereas most complexity analyses assume uniform boundedness of model Hessians, we work with potentially unbounded model Hessians. Boundedness is not guaranteed in practical implementations, in particular ones based on quasi-Newton updates. Our analysis is conducted for a family of trust-region methods that includes most known methods as special cases. We examine two regimes of Hessian growth: one bounded by a power of the number of successful iterations, and one bounded by a power of the number of iterations. This allows us to formalize and confirm the profound intuition of Powell (2010), who studied convergence under a special case of our assumptions, but whose proof contained complexity arguments. Specifically, for $0 \le p < 1$, we establish sharp $O(\epsilon^{-2/(1-p)})$ evaluation complexity to find an ϵ-stationary point when model Hessians are $O(k^p)$, where k is the iteration counter. For $p = 1$, which is the case studied by Powell, we establish a sharp $O(\exp(c\epsilon^{-2}))$ evaluation complexity for a certain constant $c > 0$. This is as Powell suspected and is far worse than other bounds surmised elsewhere in the literature. We establish similar bounds when model Hessians are $O(|S_k|^p)$, where $|S_k|$ is the number of iterations where the step was accepted, up to iteration k. To the best of our knowledge, ours is the first work to provide complexity bounds when model Hessians grow linearly with $|S_k|$ or at most linearly with k, which covers multiple quasi-Newton approximations.