Celebrating Greg Martin: A Chorus of Contributions to Analytic Number Theory
Célébration de Greg Martin : Une chorale de contributions à la théorie analytique des nombres

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STEPHEN CHOI, Simon Fraser University

Polynomials whose reducibility is related to the Goldbach conjecture

In this talk, we introduce a collection of polynomials F_N, associated to each positive integer N, whose divisibility properties yield a reformulation of the Goldbach conjecture. While this reformulation certainly does not lead to a resolution of the conjecture, it does suggest two natural generalizations for which we provide some numerical evidence. As these polynomials F_N are independently interesting, we further explore their basic properties, giving, among other things, asymptotic estimates on the growth of their coefficients.

This is a joint work with Peter Borwein, Greg Martin, and Charles Samuels.

SUSAN COOPER, University of Manitoba

A Little Support Goes A Long Way - An EDI Journey

Incorporating equity, diversity, and inclusivity (EDI) into all areas of one's life can be extremely challenging. In this talk, I will highlight how Greg Martin has contributed to important discussions at PIMS while supporting my EDI journey. I will primarily focus on concrete impacts that I have been involved in, including a summer school recently co-organized for undergraduates.

CECILE DARTYGE, Université de Lorraine, France

Exponential sums and reducible polynomials.

Abstract: Hooley proved that if f is a polynomial with integer coefficients, irreducible and with degree bigger than 2, then the fractions r/n with $0 < r < n$, n divising f(r), are uniformly distributed in]0,1[. I will present some results obtained for some reducible polynomials in a joint work with Greg Martin.

LUCILE DEVIN, Université du Littoral Côte d'Opale

Polynomial races with big ties

In the context of prime number races, Greg Martin and Nathan Ng gave a pretty weak condition under which "ties have density zero". We investigate counter-examples to this condition when we consider the analog race for polynomial with coefficients in finite fields. This is joint work with Alexandre Bailleul, Daniel Keliher and Wanlin Li.

ANDREY FEUERVERGER, University of Toronto

Statistics in Number Theory???

Bradley Efron – the widely respected professor of statistics at Stanford – is famously quoted as having once said: "Those who ignore statistics are condemned to reinvent it." But does this caution also apply to analytic number theorists? There was a time – dating back to the 1920s and 1930s, and prior to Kolmogorov's 1933 seminal work – when probability theory itself had a bad rap. And the story of how that prejudice led Hardy, Turan, and others to miss out on some spectacular results in number theory is well documented by Efron's colleague, Perci Diaconis, in his nice paper titled "G.H. Hardy and probability???, 2002, Bulletin of the London Math. Society. (The "???" in the title of our talk owes its origins to that paper.) Although some basic probability theory will be used, knowledge of statistics theory will not be required to follow this number-theoretically oriented talk. However, both the Fourier transform as well as the Riemann zeta function will each make star appearances in it. This talk is based on joint work with Greg Martin.

DANIEL FIORILLI, Université Paris-Saclay
Biases and variances in the distribution of primes

I will discuss some of my earlier work with Greg and how it has impacted the way I think about primes. In particular, I will talk about biases in the distribution of primes in arithmetic progressions, higher moments of their limiting distribution, effective central limit theorems, Diophantine properties of zeros of L-functions, and other related results.

AYLA GAFNI, University of Mississippi
Exponential Sums with Additive Coefficients

For an arithmetic function f and a real number α, consider the exponential sum

$$S_f(x,\alpha) = \sum_{n \le x} f(n) e^{2\pi i n\alpha}.$$

The growth of these sums as x increases plays an important role in many number theory techniques. We will discuss new bounds on these exponential sums for various additive functions f, including $\omega(n)$ (the number of distinct prime factors of n) and $\Omega(n)$ (the total number of prime factors of n). We will then apply these bounds to enumerate certain integer partitions and solutions to Diophantine equations. This is joint work with Nicolas Robles.

MIAO GU, University of Michigan
Factorization tests arising from counting modular forms and automorphic representations

A theorem of Gekeler compares the number of non-isomorphic automorphic representations associated with the space of cusp forms of weight k on $\Gamma_0(N)$ to a simpler function of k and N, showing that the two are equal whenever N is squarefree. We prove the converse of this theorem (with one small exception), thus providing a characterization of squarefree integers. We also establish a similar characterization of prime numbers in terms of the number of Hecke newforms of weight k on $\Gamma_0(N)$.

It follows that a hypothetical fast algorithm for computing the number of such automorphic representations for even a single weight k would yield a fast test for whether N is squarefree. We also show how to obtain bounds on the possible square divisors of a number N that has been found to not be squarefree via this test, and we show how to probabilistically obtain the complete factorization of the squarefull part of N from the number of such automorphic representations for two different weights. If in addition we have the number of such Hecke newforms for even a single weight k, then we show how to probabilistically factor N entirely. All of these computations could be performed quickly in practice, given the number(s) of automorphic representations and modular forms as input. This is joint work with Greg Martin.

MATILDE LALIN, Université de Montréal
Variances of prime independent multiplicative functions over function fields

We consider the family of multiplicative functions of $\mathbb{F}_q[T]$ with the property that the value at a power of an irreducible polynomial depends only on the exponent, but does not depend on the polynomial or its degree. We study variances of such functions in different regimes, relating them to variances of the divisor function $d_k(f)$. We consider some settings that can be related to distributions over the ensemble of unitary matrices and others related to distributions over the ensemble of unitary symplectic matrices. While most questions give very similar answers as the distributions of the divisor function, some of the symplectic problems, dealing with quadratic characters, are different and vary according to the values of the function at the square of the primes. This is joint work with Olha Zhur (Taras Shevchenko National University of Kyiv).

YU-RU LIU, University of Waterloo
Equidistribution of Polynomial Sequences in Function Fields

We prove a conjecture about Weyl's equidistribution theorem of polynomial sequences. This is joint work with Jérémy Champagne,Thái Hoàng Lê and Trevor Wooley.

AMITA MALIK, Pennsylvania State
Zeros of derivatives of L-functions attached to Maass forms

Motivated by the close connection of the zeros of the derivative of the Riemann zeta function, we study the zeros of higher order derivatives of L-function attached to Maass forms. This is joint work with Rahul Kumar.

PAUL PÉRINGUEY, University of British Columbia
Refinements of Artin's primitive root conjecture

Let $\mathrm{ord_p(a)}$ be the order of a in $(\mathbb{Z}/p\mathbb{Z})^*$. In 1927, Artin conjectured that the set of primes p for which an integer $a \neq -1, \square$ is a primitive root (i.e. $\mathrm{ord_p(a) = p - 1}$) has a positive asymptotic density among all primes. In 1967 Hooley proved this conjecture assuming the Generalized Riemann Hypothesis (GRH).

In this talk we will study the behaviour of $\mathrm{ord_p(a)}$ as p varies over primes, in particular we will show, under GRH, that the set of primes p for which $\mathrm{ord_p(a)}$ is "k prime factors away" from $p-1$ has a positive asymptotic density among all primes except for particular values of a and k. We will interpret being "k prime factors away" in three different ways, namely $k = \omega(\frac{p-1}{\mathrm{ord_p(a)}})$, $k = \Omega(\frac{p-1}{\mathrm{ord_p(a)}})$ and $k = \omega(p-1) - \omega(\mathrm{ord_p(a)})$, and present conditional results analogous to Hooley's in all three cases and for all integer k. From this, we will derive conditionally the expectation for these quantities.

Furthermore we will provide partial unconditional answers to some of these questions.

This is joint work with Leo Goldmakher and Greg Martin.

PAUL POLLACK, University of Georgia
Counting primes with a given primitive root, uniformly

I will discuss work in progress with Kai (Steve) Fan on the problem of counting primes up to x possessing a given primitive root g, uniformly in g. As a sample of our results, we show under GRH that if g is a nonsquare integer, then the least prime p having g as a primitive root is $O((\log 3|g|)^B)$ for some absolute constant B. Connections will be drawn with work done during the speaker's time as a postdoc with Greg at UBC.

REGINALD SIMPSON, University of British Columbia
The Density and Distribution of Cyclic Groups in the Invariant Factor Decomposition of the Multiplicative Group

For any integer m the multiplicative group $(\mathbb{Z}/m\mathbb{Z})^\times$ has a unique decomposition into a product of cyclic groups $\mathbb{Z}_{d_1} \times \mathbb{Z}_{d_2} \times \cdots \times \mathbb{Z}_{d_N}$ where $d_i \mid d_{i+1}$ for $1 \leq i < N$, called the invariant factor decomposition. In this talk we discuss three results from a forthcoming paper concerning this decomposition, and the techniques used to prove those results. First, for any order d: we determine the natural density of m where $\mathbb{Z}_d$ is in the invariant factor decomposition of m. Second, for any order d: we determine asymptotic formulas (in terms of some n) for how many copies of $\mathbb{Z}_d$ are there on average in the invariant factor decomposition of m in the range $1 \leq m \leq n$. Third, for any order d: we determine Erdős-Kac-like limiting distributions for the number of copies of $\mathbb{Z}_d$ in the invariant factor decomposition of the multiplicative group. The details of these results reveal a delightfully elegant pattern and the discussion of the proof techniques will demonstrate why this pattern is, when thought about carefully, intuitive.

LEE TROUPE, Mercer University
The number of subgroups of the multiplicative group

Let $\omega(n)$ denote the number of distinct prime factors of the positive integer n. According to the celebrated Erdős–Kac theorem, the values of $\omega(n)$ are, in a sense, normally distributed with mean and variance $\log\log n$. We say that an arithmetic function f satisfies an Erős–Kac law if, in the same sense, its values are normally distributed with a certain mean and variance.

Let $I(n)$ denote the number of isomorphism classes of subgroups of $(\mathbb{Z}/n\mathbb{Z})^\times$, and let $G(n)$ denote the number of subsets of $(\mathbb{Z}/n\mathbb{Z})^\times$ which are subgroups. Then $\log I(n)$ and $\log G(n)$ each satisfy an Erdős-Kac law. We will also discuss the maximal order of $\log I(n)$ and $\log G(n)$. These results date back to the speaker's time as a postdoctoral researcher at the University of British Columbia under the wise and benevolent supervision of Greg Martin.

WELCOME,

TREVOR WOOLEY, Purdue University
Smooth values of polynomials and superirreducibility

We discuss k-superirreducible polynomials, by which we mean irreducible polynomials that remain irreducible under any polynomial substitution of positive degree at most k. The existence of superirreducible polynomials with integral coefficients places constraints on potential approaches to generating smooth values of polynomials (values having only small prime factors), a topic investigated by Schinzel in 1967. We describe the motivation and background to such considerations, and report on recent work restricted to finite fields. In particular, we give an explicit formula for the number of monic 2-superirreducible polynomials having even degree d analogous to the famous formula of Gauss for the number of monic irreducible polynomials of given degree over a finite field. This talk is based on joint work of the speaker with Jonathan Bober, Lara Du, Dan Fretwell, Gene Kopp and Greg Martin.

CHI HOI YIP, Georgia Institute of Technology
Counting powerfree-like numbers

Recently, Martin, Mossinghoff and Trudgian investigated oscillation results for a family of arithmetic functions called "fake μ's". In particular, they proved oscillation results at scale $\sqrt{x}$ for a family of fake μ's. In this talk, I will discuss new oscillation results for the summatory functions of all nontrivial fake μ's, focusing on their connections with classical results on the indicator functions of powerfree numbers. Joint work with Greg Martin.

ASIF ZAMAN, University of Toronto
Improving the trivial bound for class group torsion

Let $K \neq \mathbb{Q}$ be a number field of degree $[K:\mathbb{Q}]$ and absolute discriminant $D_K = |\mathrm{Disc}(K)|$. Let Cl_K be the class group of K. For an integer $\ell \geq 2$, the ℓ-torsion of the class group of K satisfies the well-known trivial bound

$$|\mathrm{Cl}_K[\ell]| \leq |\mathrm{Cl}_K| \ll_{[K:\mathbb{Q}]} D_K^{1/2}(\log D_K)^{[K:\mathbb{Q}]-1}$$

due to Landau. Improvements over this trivial bound, both conditional and unconditional, have generated significant interest in many cases depending on ℓ, the degree $[K:\mathbb{Q}]$, and the subfield structure of K. In this talk, I will discuss an unconditional log-power savings improvement over this trivial bound for all ℓ and all number fields K. The method will be traced back to the teachings of Greg Martin.

This is joint work with Robert Lemke Oliver.