**PAUL PÉRINGUEY**, University of British Columbia

*Refinements of Artin's primitive root conjecture*

Let $\mathrm{ord_p(a)}$ be the order of $a$ in $(\mathbb{Z}/p\mathbb{Z})^*$. In 1927, Artin conjectured that the set of primes $p$ for which an integer $a \neq -1, \square$ is a primitive root (i.e. $\mathrm{ord_p(a) = p - 1}$) has a positive asymptotic density among all primes. In 1967 Hooley proved this conjecture assuming the Generalized Riemann Hypothesis (GRH).

In this talk we will study the behaviour of $\mathrm{ord_p(a)}$ as $p$ varies over primes, in particular we will show, under GRH, that the set of primes $p$ for which $\mathrm{ord_p(a)}$ is "$k$ prime factors away" from $p-1$ has a positive asymptotic density among all primes except for particular values of $a$ and $k$. We will interpret being "$k$ prime factors away" in three different ways, namely $k = \omega(\frac{p-1}{\mathrm{ord_p(a)}})$, $k = \Omega(\frac{p-1}{\mathrm{ord_p(a)}})$ and $k = \omega(p-1) - \omega(\mathrm{ord_p(a)})$, and present conditional results analogous to Hooley's in all three cases and for all integer $k$. From this, we will derive conditionally the expectation for these quantities.

Furthermore we will provide partial unconditional answers to some of these questions.

This is joint work with Leo Goldmakher and Greg Martin.