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The number of subgroups of the multiplicative group

Let $\omega(n)$ denote the number of distinct prime factors of the positive integer n. According to the celebrated Erdős–Kac theorem, the values of $\omega(n)$ are, in a sense, normally distributed with mean and variance $\log\log n$. We say that an arithmetic function f satisfies an Erős–Kac law if, in the same sense, its values are normally distributed with a certain mean and variance.

Let $I(n)$ denote the number of isomorphism classes of subgroups of $(\mathbb{Z}/n\mathbb{Z})^\times$, and let $G(n)$ denote the number of subsets of $(\mathbb{Z}/n\mathbb{Z})^\times$ which are subgroups. Then $\log I(n)$ and $\log G(n)$ each satisfy an Erdős-Kac law. We will also discuss the maximal order of $\log I(n)$ and $\log G(n)$. These results date back to the speaker's time as a postdoctoral researcher at the University of British Columbia under the wise and benevolent supervision of Greg Martin.