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Fine structure of real quadratic integer rings

For a fixed integer $D > 1$, represent the set $\mathbb{Z}(\sqrt{D})$ by the set $\mathbb{Z} \times \mathbb{Z}$. The D-norm of an element (a, b) of $\mathbb{Z} \times \mathbb{Z}$, denoted $N_D(a, b)$, is the integer $a^{2\smile}Db^2$. For each integer k, $\mathbb{Z}_k(D)$ is the k-norm class $\{(a, b) : k = N_D(a, b)\}$. For D the set $V(D) = \{k : \mathbb{Z}_k(D)$ is nonempty$\}$ is closed under integer multiplication. Each norm class $\mathbb{Z}_k(D)$ has an algebraic structure and is generated by specific elements. Moreover each of these specific generating elements produces a structural component satisfying a well-known distribution known as Benford's Law. Benford's Law is perpetuated, via algebraic properties of $\mathbb{Z} \times \mathbb{Z}$ to larger substructures of $\mathbb{Z} \times \mathbb{Z}$.

In this talk we present results on these structural aspect of the quadratic integer ring $\mathbb{Z}(\sqrt{N})$.