## ADRIANA-STEFANIA CIUPEANU, TANJIMA AKHTER, Universities of Manitoba and Alberta

Preventing HPV-Induced Cervical Cancer in Alberta, Canada: A Mathematical Modeling study

Human Papillomavirus (HPV) is a widespread sexually transmitted infection, responsible for nearly 99.7% of cervical cancer cases. Despite extensive public health efforts, controlling HPV transmission remains a challenge. This research applies a dynamic mathematical model to explore HPV infection and vaccination strategies in Alberta, Canada, aiming to identify the optimal vaccination program for both women and men to reduce infection prevalence.

A key challenge is the limited availability of reliable data for both sexes, particularly regarding prevalence. While female-specific data is abundant, male data is often overlooked in existing models, despite men playing a crucial role in HPV transmission. Addressing this gap could enhance the accuracy of models and lead to more effective public health interventions. Expanding data collection efforts to better represent males is essential for robust modelling.

The study will develop and calibrate an age- and sex-stratified mathematical model using Bayesian inference methods and MATLAB. This model will incorporate complex contact patterns and disease dynamics to simulate various vaccination scenarios and assess their long-term impacts on HPV transmission and health outcomes. The research will also estimate the potential reduction in cervical cancer cases resulting from the optimal vaccination strategy, providing quantitative evidence of its effectiveness.

Ultimately, this study aims to inform public health policy by identifying the most effective vaccination strategies for controlling HPV and preventing cervical cancer, while advocating for more comprehensive data collection to improve future modelling efforts.

Joint work with Tanjima Akhter and Michael Y Li from University of Alberta.

#### KYE EMOND, Simon Fraser University

Existence and Uniqueness for a System of a Solid in a Lorentz Gas

To accurately model the motion of an object immersed in a rarefied gas, one must construct a coupled system where the gas density evolves according to a partial differential equation with boundary conditions determined by the object motion, and the object evolves following Newton's Laws which depend on the gas density. The existence and uniqueness of solutions to this system of coupled equations is an open problem for all but one-dimensional non-interacting (ideal) gasses. We show the existence and uniqueness of a one-dimensional Lorentz gas density given arbitrary object motion. Furthermore, we show the same for the entire system of Lorentz gas and object motion assuming monotonically increasing object speed. This provides theoretical justification to the application of these models to make predictions about Lorentz gas-object interaction systems and develops strong foundations towards proving existence and uniqueness in more generality.

### JAMES HOULE, Simon Fraser University

#### Schäffer's Conjecture and the Modular Method

Schäffer's conjecture predicts that the only natural solutions to  $1^k + 2^k + \cdots + x^k = y^n$  are (x, y, k, n) = (1, 1, k, n), (24, 70, 2, 2), except when  $(k, n) \in \{(1, 2), (3, 2), (3, 4), (5, 2)\}$  in which case there are infinitely many such solutions. Bennett-Györy-Pintér have proved the conjecture for  $1 \le k \le 11$  using a combination of methods including linear forms of logarithms and the modular method.

Our goal was to see how far we could push the modular method to avoid using linear forms of logarithms. By using only the modular method in combination with an expanded set of coupled generalized Fermat equations which are derived using descent, we explain how to asymptotically solve Schäffer's conjecture for select k, additionally using the multi-Frey technique and theorem of Darmon-Merel.

#### **REX LI**, Carleton University Math Enrichment Centre Optimal Trajectories in Variable Speed Environments with Line Constraints

The research investigates optimal path selection in a 2-dimensional plane where an agent travels between two points,  $A(x_1, y_1)$ and  $B(x_2, y_2)$ , considering variable speeds on distinct trajectories. With walking speeds defined as v off the lines and kv(where k > 1) on the lines, the presence of two lines—line m with a slope of 0, and line n with a slope of  $\alpha$ , intersecting at point Z(z, 0)—adds complexity to the path optimization. This study methodically analyzes scenarios involving no use of the lines, utilization of one line, and navigation across both lines to derive travel time formulas. Each potential path's optimal entry and exit points on the lines are determined and they are later compared to each other to select the optimal path. Further exploration could be extended by increasing the number of lines available. The findings contribute to the strategic decisionmaking necessary for optimizing travel, with implications for applications in fields such as transportation planning, where the entrance and exit of the highway can be selected to minimize traveling time.

#### HAGGAI LIU, Simon Fraser University

#### Moduli Spaces of Weighted Stable Curves and their Fundamental Groups

The Deligne-Mumford compactification,  $\overline{M_{0,n}}$ , of the moduli space of n distinct ordered points on  $\mathbb{P}^1$ , has many well understood geometric and topological properties. For example, it is a smooth projective variety over its base field. Many interesting properties are known for the manifold  $\overline{M_{0,n}}(\mathbb{R})$  of real points of this variety. In particular, its fundamental group,  $\pi_1(\overline{M_{0,n}}(\mathbb{R}))$ , is related, via a short exact sequence, to another group known as the cactus group. Henriques and Kamnitzer gave an elegant combinatorial presentation of this cactus group.

In 2003, Hassett constructed a weighted variant of  $\overline{M_{0,n}}(\mathbb{R})$ : For each of the *n* labels, we assign a weight between 0 and 1; points can coincide if the sum of their weights does not exceed one. We seek combinatorial presentations for the fundamental groups of Hassett spaces with certain restrictions on the weights. In particular, we express the Hassett space as a blow-down of  $\overline{M_{0,n}}$  and modify the cactus group to produce an analogous short exact sequence. The relations of this modified cactus group involves extensions to the braid relations in  $S_n$ . To establish the sufficiency of such relations, we consider a certain cell decomposition of these Hassett spaces, which are indexed by ordered planar trees.

#### KIARA MCDONALD, University of Victoria

#### Broadcast Independence in Split Graphs

In Graph Theory, the well-known problems of domination, packing and independence are generalized by broadcast domination, broadcast packing and broadcast independence. As an analogy and application, consider a city, where one wants to place cell towers of different signal strengths subject to certain conditions. If every building in the city hears the signal from at least (respectively at most) one tower, then the broadcast is dominating (respectively packing). If no tower hears the signal from another tower, the broadcast is independent. The sum of the tower signal strengths is called the cost of the broadcast. The total cost of a maximum independent broadcast is called the broadcast independence number.

Our research was focused on determining explicit formulas and polynomial time algorithms for the broadcast independence number of various types of graphs. This parameter is difficult to compute for graphs in general, so we restrict the problem to specific classes of graphs to make use of their special structural properties. One type of graph that we examined in our research was split graphs. Split graphs are defined to have a partition of its vertices into a clique and an independent set, a property which is specific to this class of graphs. Additionally, all split graphs have diameter two or three. Using these special properties, we determined explicit formulas for the broadcast independence number of special types of split graphs. We also showed that the broadcast independence number is polynomial time solvable for all split graphs.

#### ELISE MOZZAFFARI, Kwantlen Polytechnic University

A Procedure for Obtaining a (2 + c)-Regular Graph from a Given Cycle Graph

In this project, we devise a procedure to obtain a (2 + c)-regular graph of minimum order from a given cycle graph  $C_n$ , where

 $c, n \in \mathbb{Z}^+$  and  $n \ge 3$ . We employ the use of cases to determine the minimum number of vertices that must be added to  $C_n$  such that the resulting graph R is (2 + c)-regular. The results of this project demonstrate that if  $c \le n - 3$ , then our desired graph R can be obtained by adding at most 1 vertex. Additionally, if c > n - 3, our findings indicate that R can be obtained by adding 3 + c - n vertices. Some additional results regarding the size and Hamiltonian property of R are also presented at the end of this project.

# **PRANGYA PARIDA**, University of Ottawa Cover-free families on graphs

A family of subsets of [t] is called a *d-cover-free family* (*d*-CFF) if no subset is contained in the union of any *d* others. We denote by t(d, n) the minimum *t* for which there exists a *d*-CFF of [t] with *n* subsets. t(1, n) is determined using Sperner's Theorem. For  $d \ge 2$ , we rely on bounds for t(d, n). Using the probabilistic approach, Erdös, Frankl, and Füredi proved  $3.106 \log(n) < t(2, n) < 5.512 \log(n)$ . Porat and Rothschild provided a deterministic polynomial-time algorithm to construct *d*-CFFs achieving  $t = O(d^2 \log(n))$ . Some upper bounds of t(2, n) (in some cases exact bounds) for some small values of *n* were provided by Li, van Rees, and Wei.

We extend the definition of 2-CFF to include a graph(G), called  $\overline{G}$ -CFF, where the edges of G specify the pair of subsets whose union must not cover any other subset. We denote by t(G) as the minimum t for which there exists a  $\overline{G}$ -CFF. Thus,  $t(K_n) = t(2, n)$ . We will discuss some classical results on CFFs, along with constructions of  $\overline{G}$ -CFFs. We prove that for a graph G with n vertices,  $t(1, n) \leq t(G) \leq t(2, n)$  and for an infinite family of star graphs with n vertices,  $t(S_n) = t(1, n)$ . We also provide constructions for  $\overline{P_n}$ -CFF and  $\overline{C_n}$ -CFF using a mixed-radix Gray code. This yields an upper bound for  $t(P_n)$  and  $t(C_n)$  that is smaller than the lower bound of t(2, n) mentioned above.