**REX LI**, Carleton University Math Enrichment Centre

*Optimal Trajectories in Variable Speed Environments with Line Constraints*

The research investigates optimal path selection in a 2-dimensional plane where an agent travels between two points, $A(x_1, y_1)$ and $B(x_2, y_2)$, considering variable speeds on distinct trajectories. With walking speeds defined as $v$ off the lines and $kv$ (where $k > 1$) on the lines, the presence of two lines—line $m$ with a slope of $0$, and line $n$ with a slope of $\alpha$, intersecting at point $Z(z, 0)$—adds complexity to the path optimization. This study methodically analyzes scenarios involving no use of the lines, utilization of one line, and navigation across both lines to derive travel time formulas. Each potential path's optimal entry and exit points on the lines are determined and they are later compared to each other to select the optimal path. Further exploration could be extended by increasing the number of lines available. The findings contribute to the strategic decision-making necessary for optimizing travel, with implications for applications in fields such as transportation planning, where the entrance and exit of the highway can be selected to minimize traveling time.