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Entangled states are typically incomparable

Consider a bipartite quantum system, where Alice and Bob jointly possess a pure state $|\psi\rangle$. Using local quantum operations on their respective subsystems, and unlimited classical communication, Alice and Bob may be able to transform $|\psi\rangle$ into another state $|\phi\rangle$. Famously, Nielsen's theorem provides a necessary and sufficient algebraic criterion for such a transformation to be possible (namely, the entanglement spectrum of $|\phi\rangle$ should majorise the entanglement spectrum of $|\psi\rangle$). In the same paper, Nielsen conjectured that in the limit of large dimensionality, for almost all pairs of states $|\psi\rangle, |\phi\rangle$ (according to the natural unitary invariant measure) such a transformation is not possible. That is to say, typical pairs of quantum states $|\psi\rangle, |\phi\rangle$ are entangled in fundamentally different ways, that cannot be converted to each other via local operations and classical communication.

Via Nielsen's theorem, this conjecture can be equivalently stated as a conjecture about majorisation of spectra of random matrices from the so-called trace-normalised complex Wishart–Laguerre ensemble. Concretely, let X and Y be independent $n \times m$ random matrices whose entries are i.i.d. standard complex Gaussians; then Nielsen's conjecture says that the probability that the spectrum of $XX^\dagger/\mathrm{tr}(XX^\dagger)$ majorises the spectrum of $YY^\dagger/\mathrm{tr}(YY^\dagger)$ tends to zero as both n and m grow large. We prove this conjecture, and we also confirm some related predictions of Cunden, Facchi, Florio and Gramegna.

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