# Arithmetic Aspects of Galois Representations Aspects arithmétiques des représentations de Galois

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#### RAUL ALONSO, UC Santa Barbara

An anticyclotomic Euler system for Hilbert cuspforms over a real quadratic field

Let g be a Hilbert cuspform of parallel weight over a real quadratic field F and let  $\mathrm{As}(V_g)$  denote the Asai representation associated with g. Let  $\chi$  be a Hecke character of an imaginary quadratic field K for which the  $G_K$ -representation  $\mathrm{As}(V_g)(\chi)$  is conjugate self-dual. In this talk we will explain how to construct an Euler system for  $\mathrm{As}(V_g)(\chi)$ . Expected applications include results towards the Bloch-Kato conjecture and towards an Iwasawa Main Conjecture for this representation.

This is joint work in progress with Francesc Castella, Michele Fornea and Óscar Rivero.

### ADITHYA CHAKRAVARTHY, University of Toronto

The Iwasawa  $\mu$ -invariants of elliptic curves over the rational numbers

In the 1990s, Ralph Greenberg formulated a striking conjecture about the Iwasawa  $\mu$ -invariants of elliptic curves over the rational numbers. In this talk, I will introduce this conjecture of Greenberg and discuss some recent results surrounding it.

### PAVEL COUPEK, Michigan State University

Heights of modular forms and Eisenstein congruences

Let f be an automorphic Hecke eigenform. The automorphic height  $H_{aut}(f)$  is defined in terms of the  $L^2$  norm of harmonic forms representing the singular cohomology group in which f is realized, while the arithmetic height  $H_{arit}(f)$  is a version of height of motives developed by Kato and Koshikawa. The height conjecture of Venkatesh predicts that  $H_{aut}(f)/H_{arit}(f)$  is related to the value of the adjoint L-function of f at s=1.

As the arithmetic height depends on a choice of a lattice in the Galois representation of f, it is natural to ask to what extent does the quotient  $H_{aut}(f)/H_{arit}(f)$  depend on such a choice, especially in the presence of congruences between f and an Eisenstein series. I will talk about joint work in progress with Preston Wake in this direction, in the context of classical weight 2 modular forms.

### KIM TUAN DO, UCLA

Euler systems over imaginary quadratic and biquadratic fields

Let  $f \in S_{2k}(\Gamma_0(N))$  be a newform and  $\chi$  be an anticyclotomic Hecke character of K. Let  $V_{f,\chi}$  be the Galois representation attached to f twisted by  $\chi$ . In this talk, I will describe an (anticyclotomic) Euler system over K for  $V_{f,\chi}$  with no restriction on the infinity type of  $\chi$  (the main innovation here is  $\chi$  can be an infinite order character). Here, K can represent either an imaginary quadratic field, where this case is a collaboration with F. Castella, or an imaginary biquadratic field.

Arithmetic applications include results towards the Bloch-Kato Conjecture and the (anticyclotomic) Iwasawa Main Conjecture for  $V_{f,\chi}$ .

# PAYMAN ESKANDARI, University of Winnipeg

On the unipotent parts of the Hodge and Tate conjectures

The motivic version of the Hodge conjecture for *mixed* motives (over appropriate base fields) predicts that for any mixed motive, the motivic Galois group and the Mumford-Tate group should coincide. The motivic version of the Tate conjecture

for mixed motives (again, over appropriate base fields) predicts that after a change of coefficients to  $\mathbb{Q}_{\ell}$ , the motivic Galois group of any mixed motive should be equal to the closure of the image of the absolute Galois group in the corresponding  $\ell$ -adic representation. One may consider the unipotent parts of these conjectures, i.e., the equality of the unipotent radicals of the groups in question in the case of each conjecture. In this talk, we formulate refinements of the unipotent parts of these conjectures, and give sufficient criteria in terms of End and Ext groups for these refinements to hold. The results are in the context of abstract neutral tannakian categories. In particular, the results apply to the setting of any neutral tannakian category of mixed motives and any of the fullness conjectures for enriched realizations.

# **CHI-YUN HSU**, Santa Clara University *p-adic companion forms for Yoshida lifts*

Coleman defined a p-adic theta operator on overconvergent forms, mapping forms of slope 0 and weight 2-k to forms of slope k-1 and weight k. By explicitly computing the q-expansion, he proved that the critical p-stabilization of a p-ordinary CM form lies in the image of the theta operator. The parallel statement on the Galois side is that the Galois representation of a CM form splits locally at p. Coleman and Greenberg conjectured the respective converse, but both still remain open. In the  $\mathrm{GSp}_4$  setting, the Galois representation of a Yoshida lift splits locally into two 2-by-2 blocks at p. Our goal is to prove that the critical p-stabilization of a Yoshida lift lies in the image of a relevant theta operator. With Bharathwaj Palvannan, we figured out the effect of the relevant theta operator on the q-expansion as a first step toward the goal.

### **HEEJONG LEE**, Purdue University

Recent advances on the Serre weight conjectures

The Langlands reciprocity associates a Galois representation with a certain modular form/automorphic representation. One can ask how this association reflects structures on both sides. For example, the weights of modular forms/automorphic representations are matched by the Hodge-Tate weights of Galois representations. The Serre weight conjectures are mod p analogues of this structural correspondence.

In this talk, we introduce the Serre weight conjectures and their role in the Langlands program. Then, we discuss a novel idea proving the conjecture for the group  $\mathrm{GSp}_4$  under technical assumptions using a new geometric result on Galois deformation rings. This is based on a joint work with Daniel Le and Bao Viet Le Hung.

# SIMONE MALETTO, The University of British Columbia

Congruences of special values of the symmetric square L-function

In this talk I will give a brief exposition of my upcoming thesis, generalizing work of Ray, Sujatha and Vatsal on special values of the symmetric square L-function. Then, I will show the existence of a  $\Lambda$ -adic two-variable p-adic L-function interpolating the classical complex L-function associated to the  $\chi$ -twist of the symmetric square L-function induced by an ordinary p-distinguished, non-Eisenstein modular form.

### TAM NGUYEN, UBC

Residually isomorphic modular forms and BDP p-adic L-functions

Let p>2 be a prime that is split in an imaginary quadratic field K and let  $f\in S_{2r}(\Gamma_0(N))$  be a newform whose conductor N satisfies the strong Heegner hypothesis with respect to K. In this setting, one may construct the Bertolini-Darmon-Prasanna (BDP) p-adic L-function  $L_{BDP}(f)$ . In this talk, we show a congruence between  $L_{BDP}(f_1)$  and  $L_{BDP}(f_2)$  when  $f_1$  and  $f_2$  are residually isomorphic modulo some prime power. We will also discuss some implications for the logarithms of Heegner cycles and the anticyclotomic lwasawa main conjecture.

PEIKAI QI, Michigan State University

An analogue of Greenberg pseudo-null conjecture for CM fields

We will give an analogue of Greenberg's pseudo-null conjecture for CM fields. Let K be a CM field and  $K^+$  be the unique totally real subfield of K. Assume that primes above p in  $K^+$  all splits in K. Let  $\mathfrak{P}_1,\mathfrak{P}_2,\cdots,\mathfrak{P}_s,\tilde{\mathfrak{P}}_1,\tilde{\mathfrak{P}}_2,\cdots,\tilde{\mathfrak{P}}_s$  be prime ideas in K above p, where  $\tilde{\mathfrak{P}}_i$  is the complex conjugation of  $\mathfrak{P}_i$ . We show that there is unique  $\mathbb{Z}_p$ -extension of K unramified outside  $\mathfrak{P}_1,\mathfrak{P}_2,\cdots,\mathfrak{P}_s$ . We also show that such  $\mathbb{Z}_p$ -extension for CM field has similar properties as cyclotomic  $\mathbb{Z}_p$ -extension of a totally real field. We also give some criteria for Iwasawa invariant  $\mu=\lambda=0$ . The work is joint with Matt Stokes.

### SUJATHA RAMDORAI, UBC

Iwasawa theory over anticyclotomic extensions

We give simpler and more direct proofs of the known results for the Iwasawa theory of elliptic curves over anticyclotomic extensions.

DANIEL VALLIERES, California State University - Chico

Iwasawa theory for branched  $\mathbb{Z}_p$ -towers of finite graphs

In this talk, we will explain that the analogue of Iwasawa's asymptotic class number formula in graph theory can be extended to  $\mathbb{Z}_p$ -towers of finite connected graphs that are not necessarily unramified. This is joint work with Rusiru Gambheera.

# ILA VARMA, University of Toronto

The number of  $D_4$ -fields ordered by Artin conductor

We consider the family of  $D_4$ -quartic fields ordered by the Artin conductors of the corresponding 2-dimensional irreducible Galois representations. In this talk, I will describe ways to compute the number of such  $D_4$ -quartic fields with bounded conductor. Traditionally, there have been two approaches to counting quartic fields, using arithmetic invariant theory in combination of geometry-of-number techniques, and applying Kummer theory together with L-function methods. Both of these strategies fall short in the case of  $D_4$  fields since counting quartic fields containing a quadratic subfield of large discriminant is difficult. However, when ordering by conductor, these techniques can be utilized due to additional algebraic structure that the Galois closures of such quartic fields have, arising from the outer automorphism of  $D_4$ . This result is joint work with Ali Altug, Arul Shankar, and Kevin Wilson.