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Pinned Dot Product Set Estimates

Choosing a fractal subset $A \subset \mathbb{R}^n$ and points $a, x \in \mathbb{R}^n$, the Falconer pinned dot product set problem asks how large the associated pinned dot product sets

$$\Pi_x^a(A) := \{\alpha \in \mathbb{R} : (a-x) \cdot y = \alpha, \text{ for some } y \in A\},$$

must be, relative to the Hausdorff dimension of A and choice of points $a, x \in \mathbb{R}^n$.

We discuss our new method for studying this problem. In particular, we determine lower bounds on the Hausdorff dimension of A which guarantee that $\Pi_x^a(A)$ is large in some quantitative sense for many $a \in A$. Our proof method is robust enough to show that the set $\Pi_x^a(A)$ has large Hausdorff dimension, positive measure, or nonempty interior, so long as we assume that the dimension of A is large enough. The proof utilizes both classical and recent results on orthogonal and radial projection theory. We also discuss possible extensions and open questions raised by this method.

This talk is based on upcoming joint work with S. Senger (Missouri State) and P. Bright (UBC).