Algebraic Graph Theory II
Théorie algébrique des graphes II
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ADA CHAN, York University
Pair-state transfer in distance regular graphs

In 2020, Chen and Godsil introduced pair-state transfer in the continuous-time quantum walk. In the joint work with Kim, Monterde, Ahmadi, Kirkland and Plosker, we studied a generalization called s-pair state transfer.

Let $U(t)$ be the transition matrix of the continuous-time quantum walk on a graph X, and let e_u denote the characteristic vector of a vertex u in X. We say *perfect s-pair state transfer* occurs at time τ if

$$U(\tau)\left(e_a + se_b\right) = \alpha\left(e_c + se_d\right),$$

for some vertices a, b, c, d in X. We can view perfect s-pair state transfer as the transfer of an entangled state on two qubits to another two qubits in the quantum spin network. In this talk, we focus on transfer in distance-regular graphs.

DAVID FEDER, University of Calgary
Hard-core bosons on lattices as the symmetric power of cycle graphs

The complexity for obtaining the energies and wavefunctions for n interacting bosons on a lattice with m sites is generally exponential in both n and m, underpinning recent quantum supremacy arguments for sampling the output distribution of photon interferometer arrays. Remarkably, for infinitely strong boson-boson interactions corresponding to the hard-core boson (HCB) limit, Tonks and Girardeau found an exact solution for the ground state in one dimension. But, the solution is unwieldy for the calculation of expectation values, and cannot be readily extended to excited states or to other lattices. An equivalent description of the problem in algebraic graph theory is to determine the spectrum of the nth symmetric power of the m-cycle adjacency matrix, with the maximum eigenvector corresponding to the HCB ground state. I will discuss both approaches to the HCB problem, and show that the symmetric power provides a much simpler solution as well as new insights.

CHRIS GODSIL, University of Waterloo
Continuous quantum walks on locally finite graphs.

A continuous quantum walk on a graph with adjacency matrix is determined by its transition matrix $U(t) = \exp(itA)$. We are interested in locally finite infinite graphs. The entry $(U(t))_{0,0}$ of $U(t)$ is the characteristic function of a probability distribution , the spectral density of the adjacency matrix of the graph.. I will discuss two interesting cases. For the first, the unweighted path, $(U(t))_{0,0}$ is a Bessel function, and we will see what this tells us about the continuous walk. For the second, we describe a locally finite graph (using the Poisson distribution) where the vertex 0 is periodic; this is interesting because we have shown that connected bounded infinite graphs cannot contain a periodic vertex.

GABOR LIPPNER, Northeastern University
Regular graphs with the most number of k-cycles.

We investigate which d-regular graphs have the most k-cycles per node, and find the answer for $k \leq 6$ and large enough d. Besides the usual notion of cycles, we also study variants for counting closed walks and closed non-backtracking walks. It turns out that these variants are actually easier to handle, and we can determine the answer for all k. Joint work with Arturo Ortiz San Miguel.

HERMIE MONTERDE, University of Manitoba
New results in vertex sedentariness

A vertex in a graph is said to be sedentary if a quantum state assigned on that vertex tends to stay on that vertex. In this talk, we survey new results on the topic of vertex sedentariness, provide new constructions, and discuss its connection to other types of quantum state transfer such as pretty good state transfer and uniform mixing.

SARAH PLOSKER, Brandon University
Quantum state transfer in weakly Hadamard diagonalizable graphs

Graphs whose Laplacian matrix is diagonalized by a Hadamard matrix (a $\{-1, 1\}$-valued matrix H satisfying $H^T H = nI$) have been of interest in recent years, and in particular have been studied for their quantum state transfer properties. This concept has recently been generalized to the notion of weakly Hadamard diagonalizable graphs: graphs whose Laplacian matrix is diagonalized by a $\{-1, 0, 1\}$-matrix P such that PP^T is tridiagonal. We consider quantum state transfer properties of such graphs.

LUC VINET, CRM/IVADO
Spin systems on q-hypercubes and the connection to dual polar graphs

The totally symmetric multi-qubit Dicke states arise in many quantum computing contexts. They also provide the dimensional reduction of uniform spin systems on the hypercube to the Krawtchouk chain with perfect state transfer. A q-version of these connections and their relation to the dual polar graphs will be presented.

HARMONY ZHAN, Worcester Polytechnic Institute
Limiting behavior of coined quantum walks with marked vertices

A quantum walk based search algorithm assigns to the marked vertices a special coin that incorporates the oracle. In this talk, we consider such walks where the marked vertices receive $-I$ while the unmarked vertices receive the Grover coin. We find combinatorial bases for the eigenspaces of the transition matrix, show their connection to submatrices of the adjacency matrix, Laplacian matrix and signless Laplacian matrix, and use this connection to study the limiting behavior of the walk. This is joint work with Amulya Mohan.

XIAOHONG ZHANG, Université de Montréal
Real state transfer

Let X be a graph, and let M be a Hermitian matrix associated to X, which is usually taken to be the adjacency or Laplacian matrix. At time t, the transition matrix of the continuous quantum walk on X relative to M is $U(t) = e^{itM}$. If the initial state of the walk is given by a density matrix D (positive semidefinite matrix of trace 1), then the state of the walk at time t is $D(t) = U(t)DU(-t)$. In this talk, we consider state transfer between real states (all entries of D are real), with focus on when D is rational, for example, when D is the Laplacian matrix of some graph scaled to have trace 1.