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The Fundamental Group(oid) in Discrete Homotopy Theory

Discrete homotopy theory is a homotopy theory designed for studying simple graphs, detecting combinatorial, rather than topological, “holes.” Central to this theory are the discrete homotopy groups, defined using maps out of grids of suitable dimensions. Of these, the discrete fundamental group in particular has found applications in various areas of mathematics, including matroid theory, hyperplane arrangements, and topological data analysis.

In this talk, based on joint work with C. Kapulkin (arxiv:2303.06029), we introduce the discrete fundamental groupoid and use it as a starting point to develop some robust computational techniques. A new notion of covering graphs allows us to extend the existing theory of universal covers to all graphs, and to prove a classification theorem for coverings. We also prove a discrete version of the Seifert-van Kampen theorem, generalizing a previous result of H. Barcelo et al. We then use it to solve the realization problem for the fundamental group through a purely combinatorial construction.